AP Calculus

CHAPTER 5 WORKSHEET

INTEGRALS

Name______________________________

Seat # ________ Date _______________

More Motion Problems

1. An object moving on a horizontal line has velocity $v(t) = 5\cos t$ mph in the time interval $0 \le t \le 2\pi$ hours.
   a) Find the time subintervals in which the object moves to the right, and those in which it moves to the left.
   b) Set up integral expressions to find the distance the object travels in each of these subintervals. Evaluate those expressions **without using your calculator.**
   c) Use your answer to part (b) to find the displacement and the total distance traveled in the time interval $0 \le t \le 2\pi$ hours.
   d) Set up a single integral to find the displacement in the time interval $0 \le t \le 2\pi$ hours. **Use your calculator to evaluate your integral.**
   e) Set up a single integral to find the total distance traveled in the time interval $0 \le t \le 2\pi$ hours. **Use your calculator to evaluate your integral.**

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2. An object moves on a vertical line in the time interval $0 \le t \le 16$ seconds, such that its acceleration is $a(t) = \sqrt{t}$ (ft/sec)/sec and at time $t = 0$, $v(0) = -18$ ft/sec.
   a) Find a formula for the velocity of the object at any time *t*.
   b) Set up an integral to find the displacement in the time interval $0 \le t \le 16$ seconds. **Use your calculator to evaluate your integral.**
   c) Set up an integral to find the total distance traveled in the time interval $0 \le t \le 16$ seconds. **Use your calculator to evaluate your integral.**

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3. A particle follows a linear motion in the time interval $[0, \frac{\pi}{2}]$, where time is measured in hours. The position of the particle is given by $s(t) = 8\sin t + 9$, where distances are measured in kilometers.
   a) Find the displacement in the time interval $[0, \frac{\pi}{2}]$ hours.
   b) Find a formula for the velocity of the object at any time *t*.
   c) Set up an integral to find the total distance traveled in the time interval $[0, \frac{\pi}{2}]$ hours. **Use your calculator to evaluate your integral.**
   d) Examine your answers for parts (a) and (c). What can you conclude about the motion of the particle in the time interval $[0, \frac{\pi}{2}]$ hours? Explain

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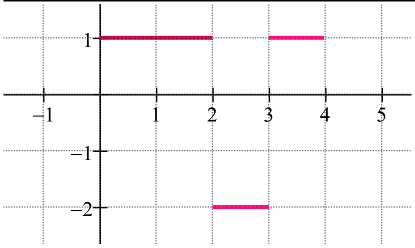
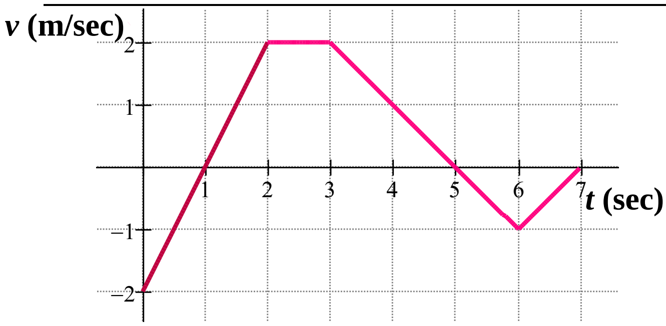
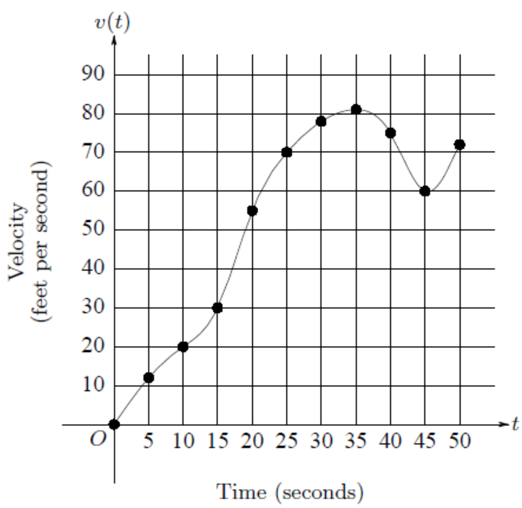


4. The graph of the velocity of a particle moving on the *x*-axis is given. The particle starts at $x = 2$ when $t = 0$.
   a) Set up an integral to find the displacement of the particle in the interval [0, 4]. **Use the graph to the left to evaluate your integral** and find the displacement.
   b) Use your answer to part (a) to find the position of the particle at time $t = 4$.
   c) Set up an integral to find the total distance traveled by the particle in the interval [0, 4]. **Use the graph provided to evaluate your integral.**



5. The graph of the velocity of a particle moving on the y-axis is given. The particle starts at $y = 3$ when $t = 0$.
   a) During what intervals of time is the acceleration of the particle positive? Explain.
   b) What is the acceleration of the particle at $t = 4$? Is the particle speeding up or slowing down? Explain.
   c) Set up an integral to find the displacement of the particle in the interval [0, 7]. **Use the graph above to evaluate your integral.** Indicate units of measure.
   d) Find where the particle is at $t = 7$. (*Hint*: use your answer from part (c)).
   e) Set up an integral expression to find the average velocity of the particle over the interval [0, 7]. **Use the graph above to evaluate your expression.** Indicate units of measure.

ESSAY FROM 1998 AB EXAM (you <u>may</u> use a graphing calculator for this question):



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

6. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.
   a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
   b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.
   c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.
   d) Approximate $\int_0^{50} v(t) \cdot dt$ with a Riemann sum, using the midpoints of five subinterval of equal length. Using correct units, explain the meaning of this integral.

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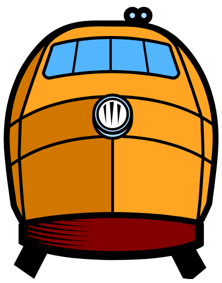
7. Mary's car runs out of gas as she is driving up a long hill. She lets the car roll without putting on the brakes. As it slows down, stops, and starts rolling backwards, its velocity up the hill is given by $v(t) = 60 - 2t$, where the velocity is measured in ft/sec and t is the number of seconds since the car run out of gas. (**Do not use your calculator** for any of these questions.)

a) When does the car stop?

b) What is the car's net displacement between $t = 10$ and $t = 40$ sec?

c) What is the total distance the car rolls between $t = 10$ and $t = 40$ sec?

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (in/sec)	0	12	22	10	5	13	11	6	2	6	0



8. The table above shows the velocity of a model train engine moving along a track for 10 seconds. Estimate the distance traveled by the train,

a) using **right endpoints** of 10 subintervals of equal length

b) using **left endpoints** of 10 subintervals of equal length

c) using the **trapezoidal rule** and 10 subintervals of equal length

d) What's special about all these answers? Why is this happening?

ANSWER KEY

More Motion Problems

1. a) Right: [0, π/2) and (3π/2, 2π]. Left: (π/2, 3π/2) (in hours)

b) $s\left(\frac{\pi}{2}\right)-s(0)=\int_0^{\frac{\pi}{2}}(5\cos t)\,dt=\sin t\Big|_0^{\pi/2}=5$ miles

$s\left(\frac{3\pi}{2}\right)-s\left(\frac{\pi}{2}\right)=\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}(5\cos t)\,dt=\sin t\Big|_{\pi/2}^{3\pi/2}=-10$ miles

$s(2\pi)-s\left(\frac{3\pi}{2}\right)=\int_{\frac{3\pi}{2}}^{2\pi}(5\cos t)\,dt=\sin t\Big|_{3\pi/2}^{2\pi}=5$ miles

c) Displacement: 0 miles; total distance: 20 miles

d) $\int_0^{2\pi}(5\cos t)\,dt=0$ miles

e) $\int_0^{2\pi}|5\cos t|\,dt=20$ miles

2. a) $v(t)=\frac{2}{3}t^{3/2}-18$

b) $s(16)-s(0)=\int_0^{16}v(t)\cdot dt=-14.933$ ft

c) Total distance $=\int_0^{16}|v(t)|\cdot dt=179.467$ ft

3. a) $s\left(\frac{\pi}{2}\right)-s(0)=8$ kilometers

b) $v(t)=8\cos t$

c) Total distance $=\int_0^{\frac{\pi}{2}}|v(t)|\cdot dt=8$ kilometers

d) Since the displacement is positive and it equals the total distance traveled, the particle moved always in the positive direction without ever switching directions.

4. a) $s(4)-s(0)=\int_0^4 v(t)\cdot dt=1$

b) $s(4)-s(0)=1 \Rightarrow s(4)=s(0)+1=3$

c) Total distance $=\int_0^4 |v(t)|\cdot dt=5$

5. a) Acceleration is positive when then velocity increases: (0, 2) and (6, 7)

b) $a(4)=-1$ m/sec^2; Since $a(4)<0$ and $v(4)>0 \Rightarrow$ Particle is slowing down

c) $s(7)-s(0)=\int_0^7 v(t)\cdot dt=3$ meters

d) $s(7)-s(0)=3 \Rightarrow s(7)=s(0)+3=6$ meters

e) average velocity = $\frac{1}{7-0}\int_0^7 v(t)\cdot dt=\frac{s(7)-s(0)}{7-0}=\frac{3}{7}$ m/sec

6. (a) Acceleration is positive on $(0,35)$ and $(45,50)$ because the velocity $v(t)$ is increasing on $[0,35]$ and $[45,50]$

3 { 1: $(0,35)$; 1: $(45,50)$; 1: reason }

Note: ignore inclusion of endpoints

(b) Avg. Acc. $=\frac{v(50)-v(0)}{50-0}=\frac{72-0}{50}=\frac{72}{50}$

or 1.44 ft/sec^2

1: answer

(c) Difference quotient; e.g.

$$\frac{v(45)-v(40)}{5}=\frac{60-75}{5}=-3 \text{ ft/sec}^2 \quad \text{or}$$

$$\frac{v(40)-v(35)}{5}=\frac{75-81}{5}=-\frac{6}{5} \text{ ft/sec}^2 \quad \text{or}$$

$$\frac{v(45)-v(35)}{10}=\frac{60-81}{10}=-\frac{21}{10} \text{ ft/sec}^2$$

–or–

Slope of tangent line, e.g.

through $(35,90)$ and $(40,75)$: $\frac{90-75}{35-40}=-3$ ft/sec^2

2 { 1: method; 1: answer }

Note: 0/2 if first point not earned

(d) $\int_0^{50} v(t)\,dt$

$\approx 10[v(5)+v(15)+v(25)+v(35)+v(45)]$

$=10(12+30+70+81+60)$

$=2530$ feet

This integral is the total distance traveled in feet over the time 0 to 50 seconds.

3 { 1: midpoint Riemann sum; 1: answer; 1: meaning of integral }

7. a) $v=0 \Rightarrow t=30\,\text{sec}$

b) $\int_{10}^{40}(60-2t)\ dt = \left(60t-t^2\right)\Big|_{10}^{40} = 300\,\text{ft}$

c) $\int_{10}^{40}\left|60-2t\right|\ dt = 500\,\text{ft}$ (graph the "V-shape" graph of the absolute value and find areas)

8. a) 87 in

b) 87 in

c) 87 in

d) All 3 approximations yield the same value because the function is equal to zero both at the beginning and end of the interval.