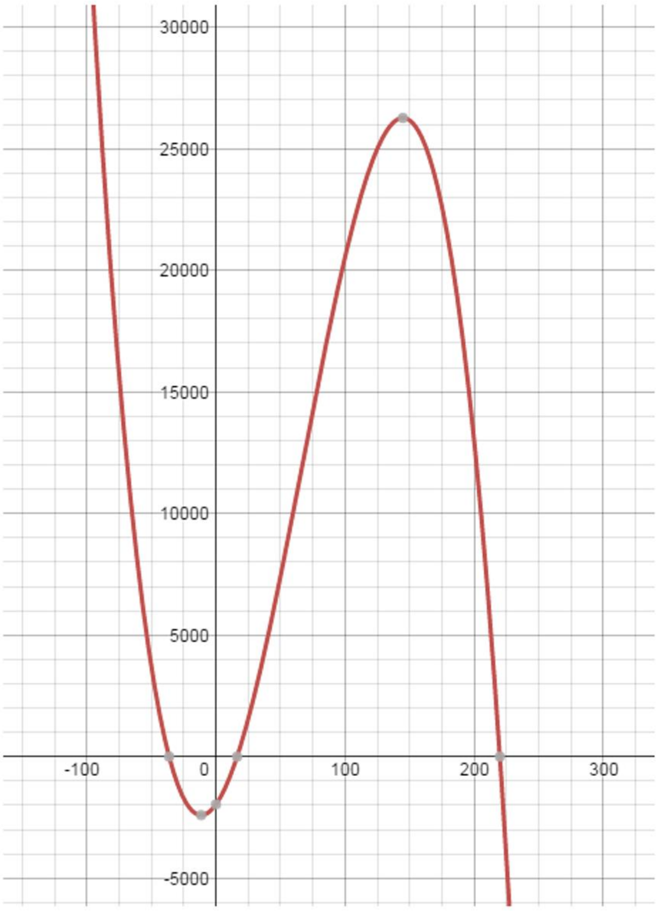
MODELLING WITH CUBIC FUNCTIONS

WORKSHEET 1

QUESTION 1

The profit model of a manufacturer is $P(x) = 75x + 3x^2 - 0.015x^3 - 1960$ where x is the number of toys that are produced.



(a) What is the break-even production level?

(b) What is the maximum profit?

(c) What is the domain of this function?

(d) What levels of production does this manufacturer lose money?

(e) What are the manufacturer's fixed costs?

QUESTION 2

An open box with a square base has a volume of 15 ft^3.

(a) Write a function that models the surface area of the box (as a function of the base side (x)).

(b) What is the domain of x?

(c) What dimensions would minimise the amount of material needed to build the box? State dimensions correct to 2 decimal places.

(d) What is the surface area of the minimised box? State your answer correct to 2 decimal places.

QUESTION 3
The volume of a metal box is 30 cubic metres. If the length is 5 metres greater that the height and the width is 2 metres less than the height, what are the dimensions of the box?

Solution

QUESTION 4
A manufacturer cuts squares from the corners of a 10cm by 16cm piece of sheet metal and then folds the metal to make an open-top box.

(a) Find an equation of the volume of the box in terms of x, V(x).

(b) What is the domain of the function?

(c) What is the maximum volume of the box?

(d) Find the height of the box if the volume must equal 78 cubic cm. State your answer correct to 3 decimal places.

Solution

QUESTION 5
A particular storage bin is cube shaped. If the side dimension is reduced by 4.5cm, the volume of the bin will be reduced by 85000cm^3. Determine the side dimension of the original cube, correct to 1 decimal place.

Solution

QUESTION 6
A rectangular box has dimensions 4m by 3m by 4m. Increasing each dimension of the box by the same amount yields a new box with volume four times the old. How much was each dimension on the original box increased to create the new box? State your answer correct to two decimal places.

Solution

QUESTION 7
In a rectangular piece of cardboard with perimeter 20ft, three parallel and equally spaced creases are made. The cardboard is then folded to make a rectangular box with open square ends. Show that the volume of the box is $V = x^2(10-4x)$.

Solution

QUESTION 8

In a rectangular piece of cardboard with perimeter 30in, two parallel and equally spaced creases are made. The cardboard is then folded to make a prism with open ends that are equilateral triangles. Show that the volume of the prism is $V(x)=\dfrac{\sqrt{3}}{4}x^2(15-3x)$.

Solution

QUESTION 9

Irena would like to make an open-top box from a square piece of tin whose side length is a. She cuts out equal squares from the corners and then folds up the tin to form the sides. What length should the cut-out squares be if the box is to have the greatest possible volume?

Solution

SOLUTIONS

QUESTION 1

(a)

Break even point = X intercepts = (16.3, 0)

ie. Starts earning money at 17 toys

(b)

Maximum profit occurs at the local maximum

26, 261

(c)

$[0, 220]$

(d)

$\{x: 0 < x < 17\} \cup \{x: x > 220\}$

(e)

\$1960 is the fixed cost as this is the amount the manufacturer loses if it produces 0 toys.

QUESTION 2

(a)

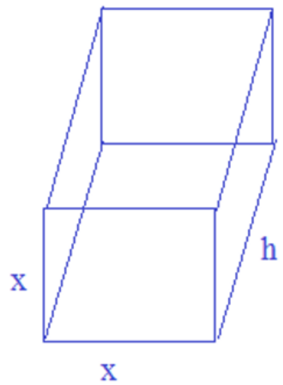
$V = lwh = x^2 h = 15$

$\therefore h = \frac{15}{x^2}$

$SA = x^2 + 4xh$

Substitute $h = \frac{15}{x^2}$:

$SA = x^2 + 4x\left(\frac{15}{x^2}\right) = x^2 + \frac{60}{x}$



(b) $0 < x < 15$

(c)

Using CAS, surface area is at a minimum when

$x = 3.11$

$\therefore h = \frac{15}{(3.11)^2} = 1.55$

Dimensions are $3.11 \times 3.11 \times 1.55$ feet

Alternatively:

$SA'(x) = 2x - \frac{60}{x^2}$

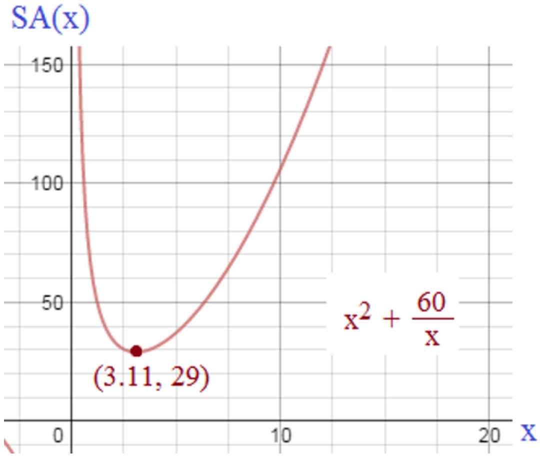
Let $SA'(x) = 0$

$2x - \frac{60}{x^2} = 0$

$2x^3 = 60$

$x^3 = 30$

$x = \sqrt[3]{30} = 3.11$ feet



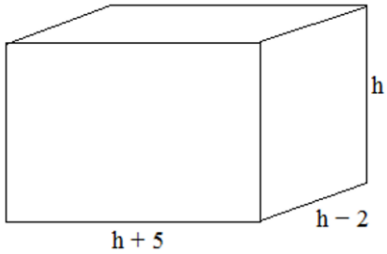
(d)

Using CAS, minimum surface area is 28.96 feet^2

OR

$SA\left(\sqrt[3]{30}\right) = \left(\sqrt[3]{30}\right)^2 + \frac{60}{\left(\sqrt[3]{30}\right)} = 28.96 \text{ ft}^2$

QUESTION 3



$V = L \times w \times h = 30$

$\therefore (h+5)(h-2)h = 30$

$(h+5)(h^2-2h) = 30$

$h^3 - 2h^2 + 5h^2 - 10h = 30$

$h^3 + 3h^2 - 10h - 30 = 0$

$h^2(h+3) - 10(h+3) = 0$

$(h^2-10)(h+3) = 0$

$h = \pm\sqrt{10}, -3$

As dimensions can't be negative, $h = \sqrt{10}$

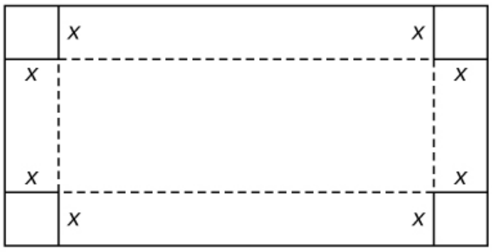
Dimensions of box are : $h = \sqrt{10}$

$L = \sqrt{10} + 5$

$w = \sqrt{10} - 2$

QUESTION 4

(a)



$V = L \times w \times h$

$= (16-2x)(10-2x)(x)$

$= (160 - 32x - 20x + 4x^2)x$

$= 160x - 32x^2 - 20x^2 + 4x^3$

$V = 160x - 52x^2 + 4x^3$

(b) $0 < x < 5$

(c)

$$\frac{dV}{dx} = 160 - 104x + 12x^2 = 0$$

$$x = 2, \frac{20}{3}$$

As $\frac{20}{3}$ lies outside the possible domain,

$x = 2$.

$$V(2) = 160(2) - 52(2)^2 + 4(2)^3 = 144 \text{ m}^3$$

(d)

Find x when $V = 78$

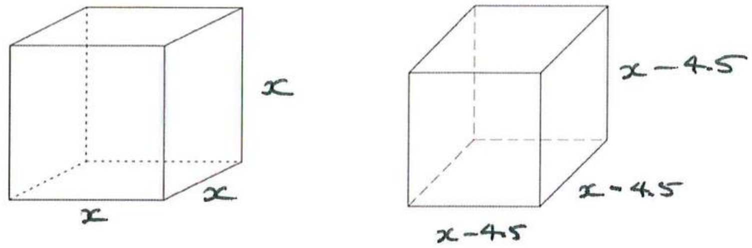
$$160x - 52x^2 + 4x^3 = 78$$

$$4x^3 - 52x^2 + 160x - 78 = 0$$

$$x = 0.599, 3.777, ~~8.624~~$$

when $x = 0.599$ m or 3.777 m

QUESTION 5



$V_1 = x^3$

$V_2 = (x-4.5)^3$

$V_2 = V_1 - 85,000$

$= x^3 - 85,000$

Equating V_2 gives:

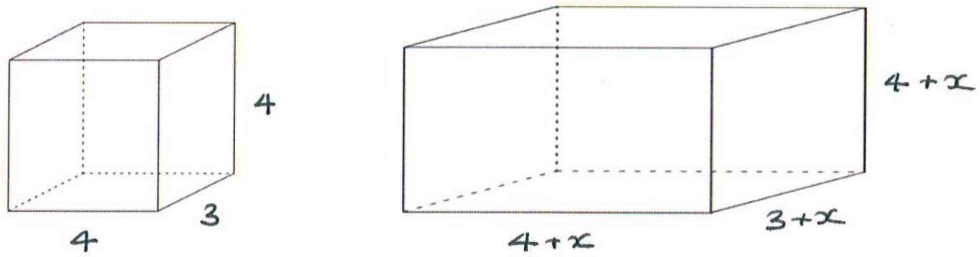
$(x-4.5)^3 = x^3 - 85,000$

$x = -77.0886\ ,\ 81.5886$

As dimensions can't be negative,

$x = 81.5886 = 81.6$ cm

QUESTION 6



Increase each dimension by x metres

$V_{original} = lwh = 4 \times 3 \times 4 = 48\ m^2$

$V_{new} = 4 \times V_{original} = 4 \times 48 = 192\ m^2$

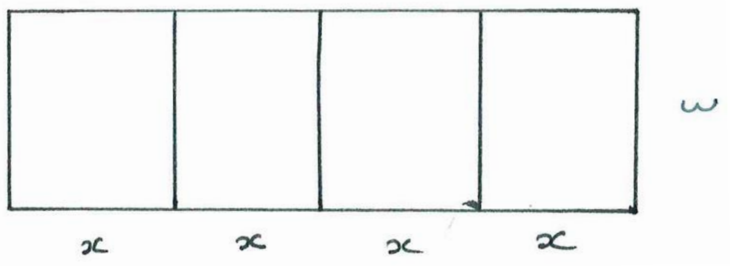
$V_{new} = l \times w \times h = 192$

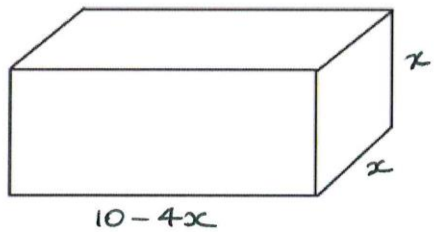
$(4+x)(3+x)(4+x) = 192$

$x = 2.12233$ m

$x = 2.12$ m

QUESTION 7





$$L + w + L + w = 20$$

$$\therefore 2L + 2w = 20$$

$$\therefore L + w = 10$$

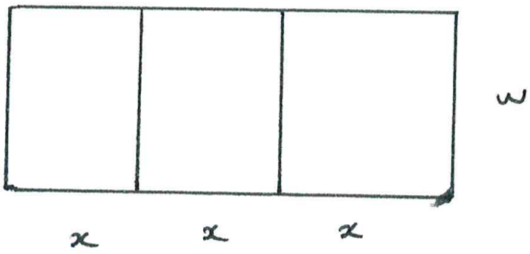
$$\therefore 4x + w = 10$$

$$\therefore w = 10 - 4x$$

$$V = Lwh = x \times x \times (10 - 4x)$$

$$= x^2(10 - 4x)$$

QUESTION 8



$L + w = 15$

$\therefore L = 15 - 3x$

$c^2 = a^2 + b^2$

$a^2 = c^2 - b^2$

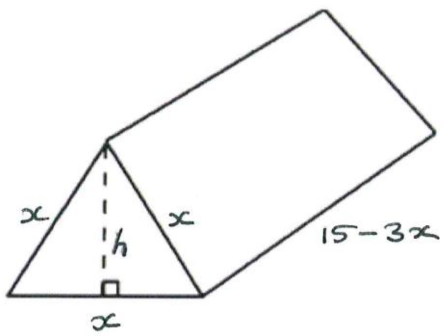
$= x^2 - \left(\frac{x}{2}\right)^2 = \frac{3x^2}{4}$

$\therefore a = h = \frac{x\sqrt{3}}{2}$

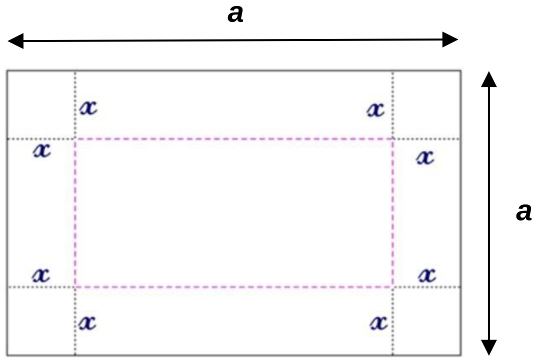
$V = \frac{1}{2} \times b \times h \times L$

$V = \frac{1}{2} \times x \times \frac{x\sqrt{3}}{2} \times (15 - 3x)$

$V = \frac{\sqrt{3}}{4} x^2 (15 - 3x)$



QUESTION 9



Let x = length of cut-out square

Length of tin box = $a - 2x$

width of tin box = $a - 2x$

$V = (a-2x)^2 x$

$= (a^2 - 2ax - 2ax + 4x^2)x$

$= (a^2 - 4ax + 4x^2)x$

$= a^2x - 4ax^2 + 4x^3$

$\frac{dV}{dx} = a^2 - 8ax + 12x^2 = 0$

$x = \frac{a}{2}, \frac{a}{6}$

when $x = \frac{a}{2}$, all the tin would be cut away,

therefore, $x = \frac{a}{6}$ gives the maximum volume.

$\therefore$ The side of the square to be cut out is one-sixth of the side of the given square.