

Worksheet #1: Graphing Exponential Functions

1. Graph each of the following functions:

a) $y = 2^x$

b) $y = 2^{x+2}$

c) $y = 2^{x-3}$

2. Graph each of the following functions:

a) $y = 3^{-x}$

b) $y = 3^{2-x} + 2$

c) $y = -3^x - 1$

3. Graph each of the following functions:

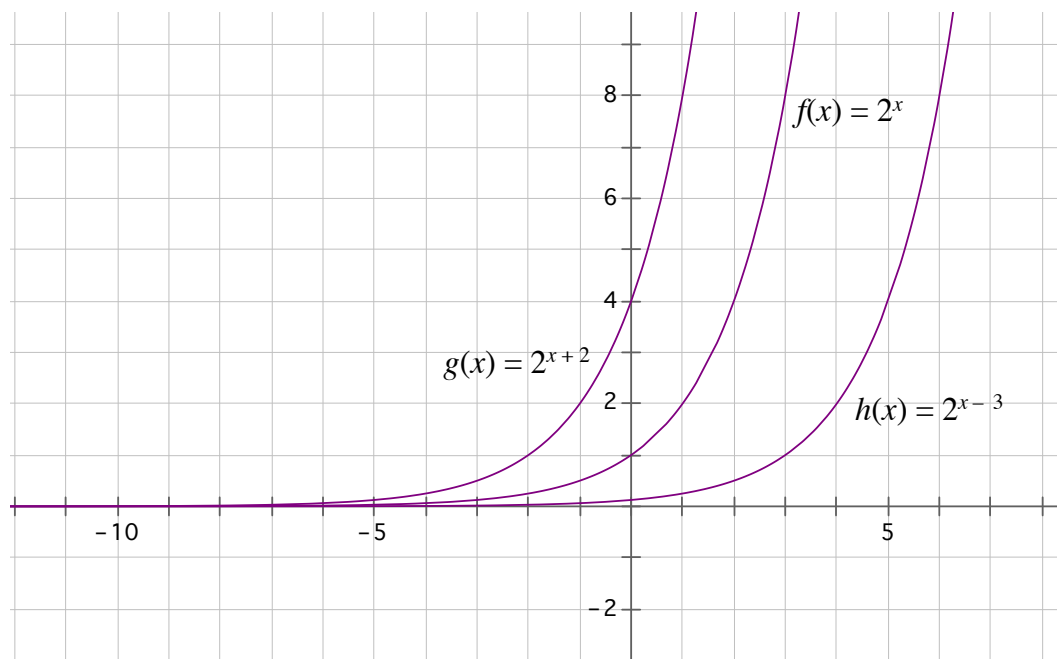
a) $y = \left(\frac{1}{2}\right)^{x-2} - 3$

b) $y = -2^{3-x} + 3$

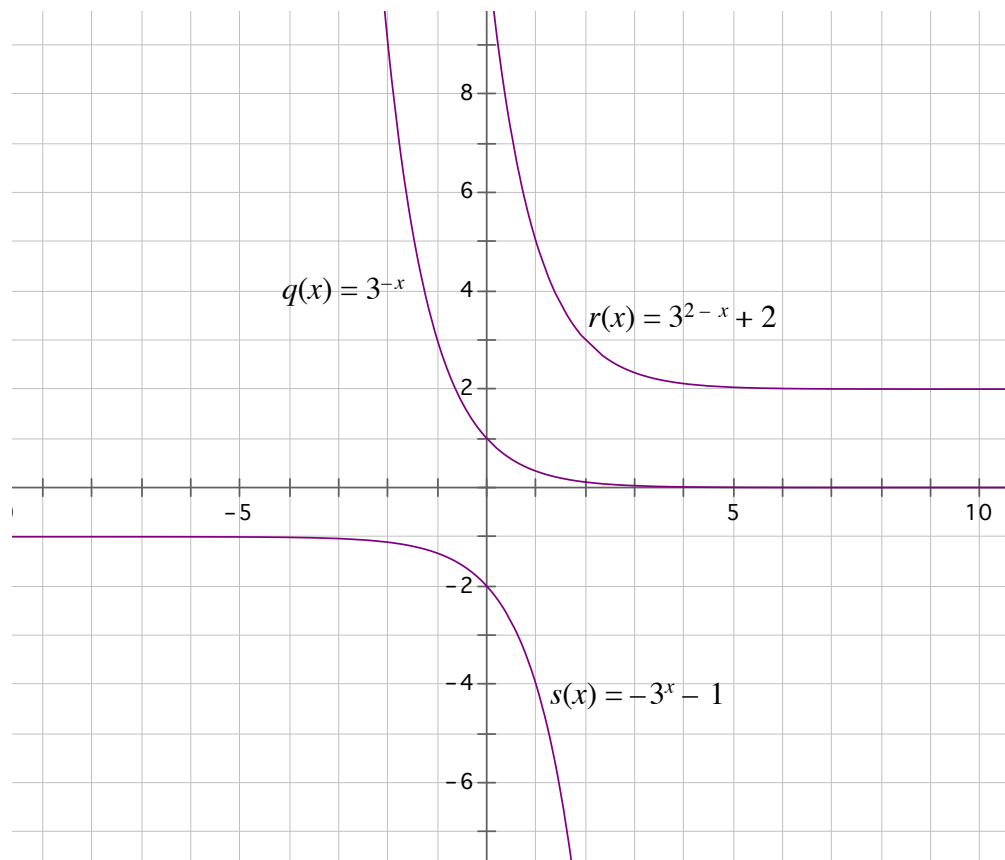
c) $y = -2 \cdot 2^{x+1}$

Answers:

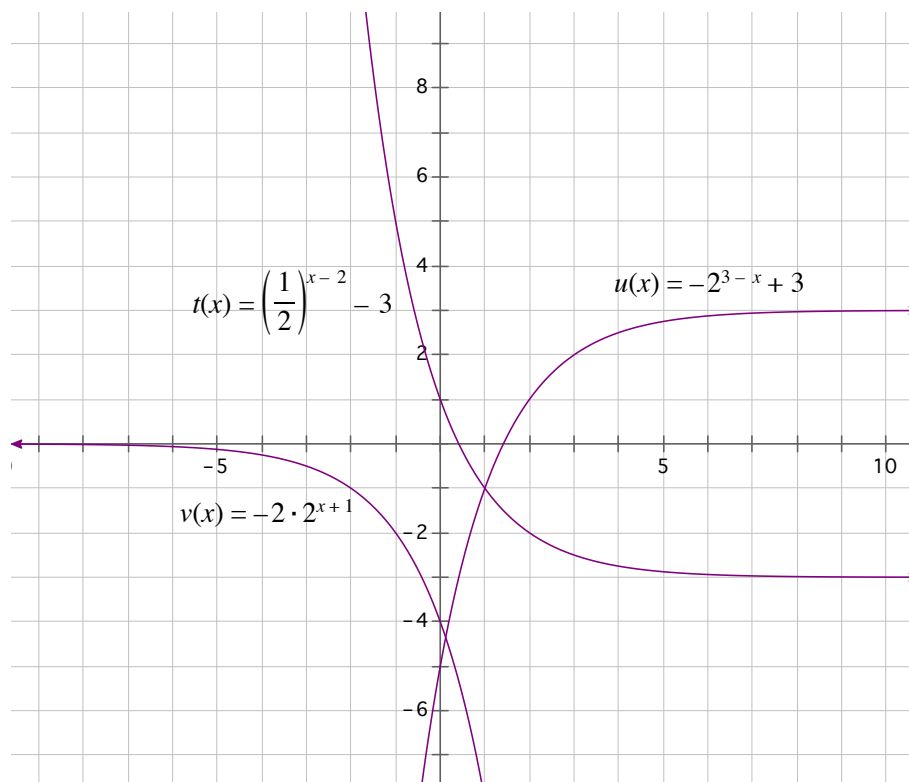
1.



2.



3.



Worksheet #2: Graphing Logarithmic Functions

1. Graph each of the following functions:

a) $y = \log_2(x + 2)$

b) $y = \log_2(3 - x)$

c) $y = \log_2(x - 1) + 3$

2. Graph each of the following functions:

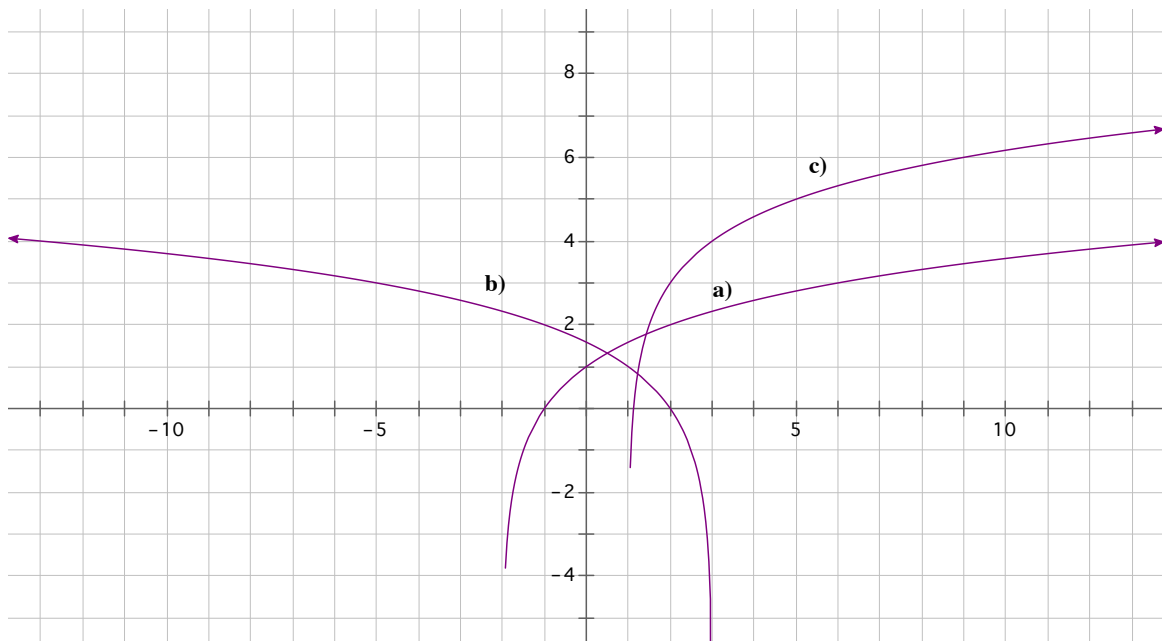
a) $y = -\log_2 x + 2$

b) $y = 2\log_2 x + 1$

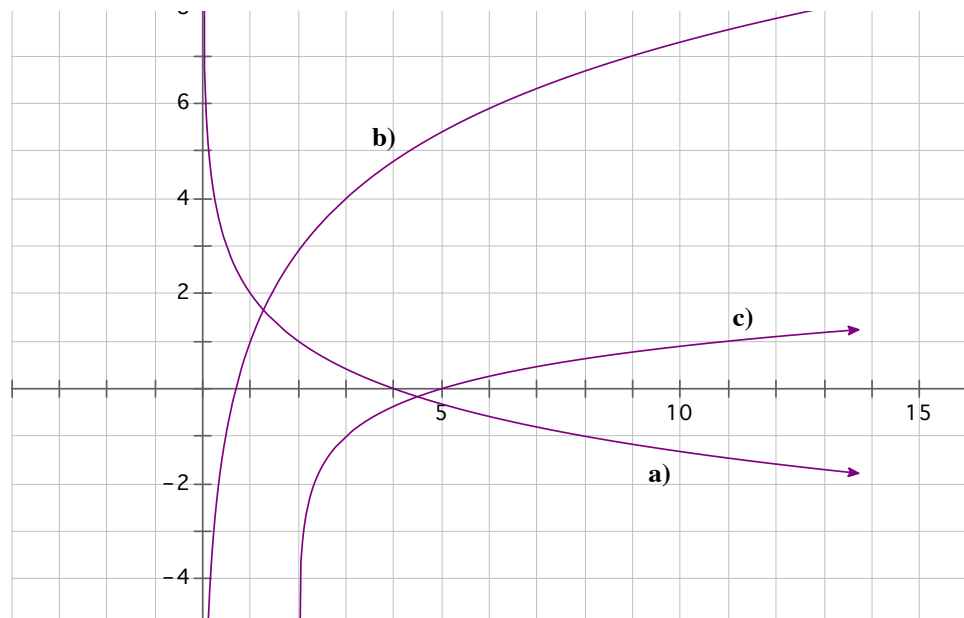
c) $y = \log_3(x - 2) - 1$

Answers:

1.



2.



Worksheet #3 – More Graphing Practice of Exponential and Logarithmic Functions

1. Graph the following functions:

a) $f(x) = -\frac{1}{2} \cdot 2^{x+2}$

b) $g(x) = -\log_2(2-x)$

2. Graph the following functions:

a) $h(x) = 3 \cdot \left(\frac{1}{2}\right)^{1-x} + 2$

b) $k(x) = 2 \log_3(x+4) - 1$

3. Determine the inverse of the following functions:

a) $f(x) = 3^{x+4} - 1$

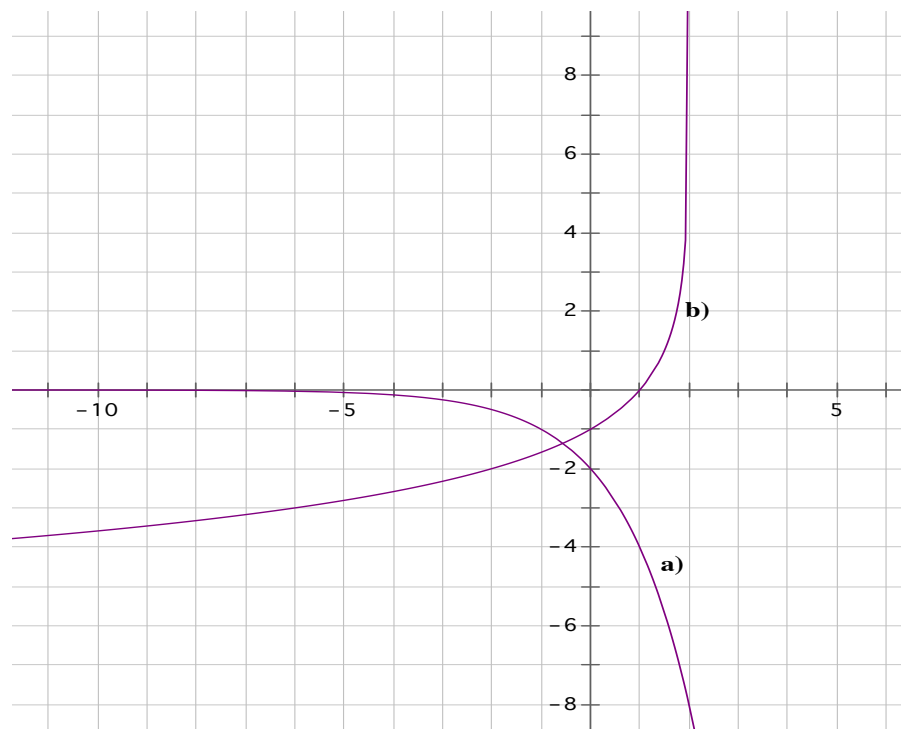
b) $g(x) = -3 \cdot 2^{1-x} + 4$

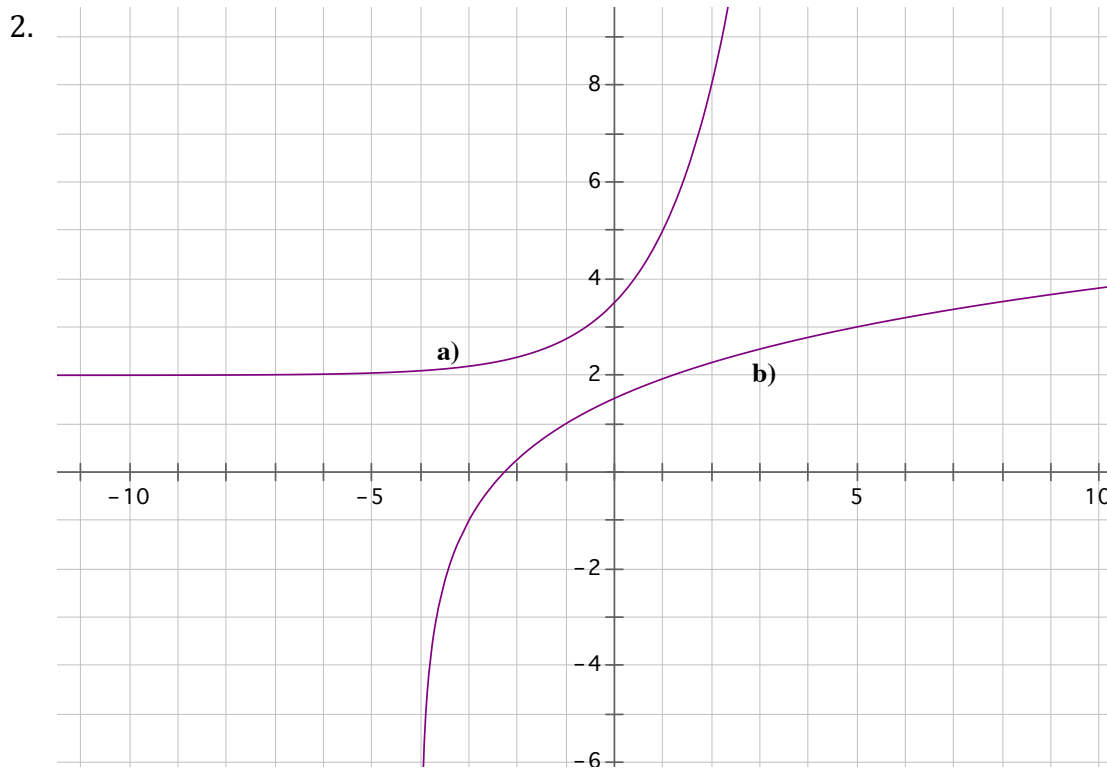
c) $h(x) = \log_3(x+2)$

d) $k(x) = -2 \log(x-4) - 5$

Answers:

1.





3. a) $f^{-1}(x) = \log_3(x+1) - 4$

b) $g^{-1}(x) = 1 - \log_2\left(\frac{x-4}{-3}\right)$

c) $h^{-1}(x) = 3^x - 2$

d) $k^{-1}(x) = 10^{\left(\frac{x+5}{-2}\right)} + 4$

Worksheet #4 – Exponential Expressions and Equations**Lesson 4.2**

1. Write as a single power. Express your answers with positive exponents.

a) $5(5^4)^{-4}$ d) $\frac{3(3)^6}{3^5}$
 b) $\frac{(-8)^4}{(-8)^5}$ e) $\left(\frac{1}{10}\right)^6 \left(\frac{1}{10}\right)^{-4}$
 c) $(9^3)^6$ f) $\left(\frac{(7)^2}{(7)^4}\right)^{-1}$

2. Evaluate. Express answers in rational form.

a) $4^{-2} - 8^{-1}$ c) $25^{-1} + 3(5^{-1})^2$
 b) $(4 + 8)^0 - 5^{-2}$ d) $\left(-\frac{1}{2}\right)^3 + 4^{-3}$

3. Evaluate. Express answers in rational form.

a) $\left(\frac{4}{7}\right)^2$ c) $\left(\frac{-2}{3}\right)^{-3}$
 b) $\left(-\frac{2}{5}\right)^3$ d) $\frac{(-3)^{-2}}{(-3)^{-5}}$

Lesson 4.3

4. What restrictions are there on the value of x in x^{-1} ? Are these restrictions different for $x^{\frac{1}{2}}$? Explain.

5. Evaluate. Express answers in rational form.

a) $\left(\frac{49}{81}\right)^{\frac{1}{2}}$ d) $\left((-125)^{\frac{1}{3}}\right)^{-3}$
 b) $\sqrt{\frac{100}{121}}$ e) $\sqrt[4]{(-9)^{-2}}$
 c) $\left(\frac{16}{9}\right)^{-0.5}$ f) $\frac{-\sqrt[3]{512}}{\sqrt[5]{-1024}}$

7. Evaluate. Express answers to three decimals.

a) $-456^{\frac{4}{7}}$ c) $\left(\frac{5}{8}\right)^{\frac{2}{8}}$
 b) $98^{0.75}$ d) $(\sqrt[5]{-1000})^3$

8. Evaluate $-8^{\frac{4}{3}}$ and $(-8)^{\frac{4}{3}}$. Explain the difference between the two.

Lesson 4.4

9. Simplify. Express answers with positive exponents.

a) $\frac{(x^{-3})x^5}{x^7}$ d) $\frac{(-2x^5)^3}{8x^{10}}$
 b) $\frac{(n^{-4})n^{-6}}{(n^{-2})^7}$ e) $(3a^2)^{-3}(9a^{-1})^2$
 c) $\left(\frac{(y^2)^6}{y^9}\right)^{-2}$ f) $\frac{(4r^{-6})(-2r^2)^5}{(-2r)^4}$

10. Simplify. Express answers with positive exponents.

a) $\frac{x^{0.5}y^{1.8}}{x^{0.3}y^{2.5}}$ d) $\left(\frac{2abc^3}{(2a^2b^3c)^2}\right)^{-2}$
 b) $\frac{(mn^3)^{-\frac{1}{2}}}{m^{\frac{1}{2}}n^{-\frac{5}{2}}}$ e) $\frac{\sqrt[4]{81p^8}}{\sqrt{9p^4}}$
 c) $\frac{\sqrt{x^2y^4}}{(x^{-2}y^3)^{-1}}$ f) $\frac{\sqrt[6]{(8x^6)^2}}{\sqrt[4]{625x^8}}$

11. Evaluate each expression for $a = 2$ and $b = 3$. Express values in rational form.

a) $\left(\frac{b^3}{a^{\frac{5}{2}}}\right)^2 \left(\frac{2a^4}{b^5}\right)$ b) $\sqrt{\frac{9b^3(ab)^2}{(a^2b^3)^3}}$

12. Simplify.

a) $(a^{10+2p})(a^{-p-8})$
 b) $(2x^2)^{3-2m} \left(\frac{1}{x}\right)^{2m}$
 c) $[(c)^{2n-3m}](c^3)^m \div (c^2)^n$
 d) $(x^{4n-m}) \left(\frac{1}{x^3}\right)^{m+n}$

Answers:

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1. a) 5^5 c) 9^{18} e) $\left(\frac{1}{10}\right)^2$
 b) $\left(-\frac{1}{8}\right)^1$ d) 3^2 f) 7^2
2. a) $-\frac{1}{16}$ b) $\frac{24}{25}$ c) $\frac{4}{25}$ d) $-\frac{7}{64}$
3. a) $\frac{16}{49}$ b) $-\frac{8}{125}$ c) $-\frac{27}{8}$ d) -27
4. x cannot be zero or a negative number for $x^{\frac{-1}{2}}$. x can be zero for $x^{\frac{1}{2}}$, but not negative.
5. a) $\frac{7}{9}$ c) $\frac{3}{4}$ e) $\frac{1}{3}$
 b) $\frac{10}{11}$ d) $-\frac{1}{125}$ f) 2
7. a) -33.068 b) 31.147 c) 0.745 d) -63.096
8. $-8^{\frac{4}{3}} = -(\sqrt[3]{8})^4 = -(2)^4 = -16$ and $(-8)^{\frac{4}{3}} = (\sqrt[3]{-8})^4 = (-2)^4 = 16$
 The second expression has an even root so the negative sign is eliminated.
9. a) $\frac{1}{x^5}$ c) $\frac{1}{y^6}$ e) $\frac{3}{a^8}$
 b) n^4 d) $-x^5$ f) -8
10. a) $\frac{x^{0.2}}{y^{0.7}}$ c) $\frac{y^5}{x}$ e) 1
 b) $\frac{n}{m}$ d) $\frac{4a^6b^{10}}{c^2}$ f) $\frac{2}{5}$
11. a) $\frac{2b}{a} = 3$ b) $\frac{3}{a^2b^2} = \frac{1}{12}$
12. a) a^{p+2} b) $2^{3-2m}x^{6-6m}$ c) 1 d) x^{n-4m}

Worksheet #5-Exponential Equations

1. Solve the following equations for x .

a) $16 \times 8^{1-x} = 1$

b) $2 \times 5^{2x-9} = 250$

c) $2^{2x} - 2^{x+1} = 8$

d) $5^{2x} = 30(5^x) - 125$

2. Solve for x :

a) $4^x - 9(2^x) = -8$

b) $25^x - 3(5^x) - 10 = 0$

c) $4^x + 4^{-x} = \frac{5}{2}$

d) $4^x + 4^{x+1} = 40$

3. Solve each of the following for the variable indicated:

a) $4^x \times 8 = \left(\frac{1}{2}\right)^x$

b) $5^{x+1} + 5^x + 5^{x-1} = \frac{31}{25}$

c) $4^x - 16^{x+1} = -62$

d) $9^x - 2(9^{-x}) = \frac{7}{3}$

e) $5^{2x+1} - 5^x = 120$

f) $8^{x-\frac{1}{3}} + 8^{x+\frac{2}{3}} = 18$

Answers:

1. a) $\frac{7}{3}$

b) 6

c) 2

d) 1 and 2

2a) 3, 0

b) 1

c) $-\frac{1}{2}, \frac{1}{2}$

d) $\frac{3}{2}$

3a) -1

b) -1

c) $\frac{1}{2}$

d) $\frac{1}{2}$

e) 1

f) $\frac{2}{3}$

Worksheet #6 - Change of Base

Calculators are to be used for question ONE and only for calculations for question FIVE (can't use log function).

1. Determine the value of each of the following, correct to three decimal places.

a) $\log_5 21$

b) $\log_7 124$

2. Simplify each of the following using change of base rules:

a. $\log_{32} 2$

b. $\log_9 3$

c. $\log_{81} 3$

d. $\log_{100} \frac{1}{1000}$

e. $\log_{25} 625$

f. $\log_{\frac{1}{4}} 2$

3. Simplify the following expression: $\log_{n^2} n$, assuming that n is positive.

4. Using the change of base formula, evaluate:

$$\log_2 8 \log_8 16 \log_{16} 32 \log_{32} 64 \log_{64} 128$$

5. Given that $\log_{10} 2 = .30103$, $\log_{10} 7 = .84509$, find $\log_7 \sqrt{2}$.

6. Prove that the following statements are true.

a) $\frac{1}{\log_3 a} + \frac{1}{\log_3 a} = \frac{1}{\log_{15} a}$

b) $(\log_a b)(\log_b a) = 1$

c) $\frac{2}{\log_8 a} - \frac{4}{\log_2 a} = \frac{1}{\log_4 a}$

Answers:

1. a) 1.892 b) 2.477

2. a) $\frac{1}{5}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $-\frac{3}{2}$ e) 2 f) $-\frac{1}{2}$

3. $\frac{1}{2}$

4. 7

5. 0.178

6. Different methods – take up in class

Worksheet #7 – Exponential/Logarithmic Applications

1. A product of nuclear explosion is the plutonium-239, which has a half-life of 24 000 years. What percent of plutonium-239 remains after 1000 years?
2. The population of a colony of bacteria can double in 25 min. After one hour, how many times as great is the population as it was after 25 min.?
3. On July 26, 1986, an earthquake with magnitude 5.5 hit California. The next day a second earthquake with magnitude 6.2 hit the same region. How many times as intense as the first earthquake was the second earthquake?
4. In 1985/86 three earthquakes hit Mexico City. How many times as intense as:
 - a) the 2nd earthquake was the 1st?
 - b) the 3rd earthquake was the 2nd?
 - c) The 3rd earthquake was the 1st?

Mexico City Earthquakes	
Date	Magnitude
Sept. 19, 1985	8.1
Sept. 21, 1985	7.5
April 30, 1986	7.0

5. Between 1956 and 1976 the annual average pH of precipitation at Sault Ste. Marie, Ontario, dropped from 5.6 to 4.3. How many times as acidic as the precipitation in 1956 was the precipitation in 1976?
6. In the spring, the pH of a stream dropped from 6.5 to 5.5 during a 3-week period in April.
 - a) How many times as acidic did the stream become?
 - b) Why would this happen in April?
 - c) The mean pH of Lake Huron is 8.2. How many times as acidic was the stream:
 - i) before the 3-week period?
 - ii) after the 3-week period?
7. When the pH of the water in a lake falls before 4.7, nearly all species of fish in the lake are deformed or killed. How many times as acidic as clean rainwater, which has a pH of 5.6, is such a lake?

Answers:

1. 97% 2. 2.64 3. 5.012
 4. a) 3.98 b) 3.16 c) 12.59 5. 19.95
 6. a) 10 b) more rain in spring c) (i) 50.12 (ii) 501.12 7. 7.94

Review Worksheet #1**Logarithms and Exponentials**

1. Evaluate:

a) $\log_{10} 1000$

b) $\log_4 1$

c) $\log_3 27$

d) $\log_2 \frac{1}{4}$

e) $\log_a a^x$

2. Solve for x .

a) $\log_4 x = 2$

b) $\log_{\frac{1}{3}} x = 4$

c) $\log_{10}(2x+1) = 2$

d) $\log_2 64 = x$

e) $\log_b 81 = 4$

3. a) Use log laws to solve $\log_3 x = \log_3 7 + \log_3 3$.b) Without tables, simplify $2\log_{10} 5 + \log_{10} 8 - \log_{10} 2$.c) If $\log_{10} 8 = x$ and $\log_{10} 3 = y$, express the following in terms of x and y only:

i. $\log_{10} 24$

ii. $\log_{10} \frac{9}{8}$

iii. $\log_{10} 720$

Solutions:

1 a) 3

b) 0

c) 3

d) -2

e) x

2 a) 16

b) $\frac{1}{81}$

c) 49.5 or $\frac{99}{2}$

d) 6

e) 3

3a) 21

b) 2

c) i. $x+y$

ii. $2y-x$

iii. $2y+x+1$

Review Worksheet #2**Logarithms and Exponential Review**

1. Evaluate each logarithm.

a) $\log_2 16$

b) $\log_2 4$

c) $\log_3 27$

d) $\log_3 1$

e) $\log_5 125$

f) $\log_5 \frac{1}{25}$

g) $\log_2 \frac{1}{8}$

h) $\log_2 (8)^5$

i) $\log_4 2$

2. Solve. (Hint: rewrite as an exponential equation.)

a) $\log_3 x = -2$

b) $\log_2 x = 4$

c) $\log_5 x = 2$

d) $\log_{16} x = \frac{1}{4}$

e) $\log_{27} x = \frac{1}{3}$

f) $\log_2 8 = x$

g) $\log_2 x = 9$

h) $\log_{\sqrt{2}} 32 = x$

i) $\log_x 16 = 2$

j) $\log_3 x = \frac{1}{2}$

k) $\log_x 125 = 3$

l) $\log_8 \frac{1}{4} = x$

m) $\log_x 3 = \frac{1}{2}$

n) $\log_{\frac{1}{3}} 9 = x$

3. Evaluate.

a) 8^{-2}

b) $8^{\frac{1}{3}}$

c) $9^{\frac{3}{2}}$

d) -7^0

e) $(3^2 - 2^2)^{-2}$

f) $\left(\frac{2}{3}\right)^{-2}$

4. Simplify the following expressions, leaving your final answer with **positive exponents**.

a) $(-2x^2y^3)^4$

b) $\left(\frac{3k^5}{k^2p}\right)^{-3}$

c) $\sqrt[3]{125x^{12}} \times \sqrt{16x^8}$

d) $\frac{(3x^2y^5)^{-2}}{8x^{-3}y^{-3}}$

5. Solve.

a) $9^{x-1} \times \frac{1}{3^{1-4x}} = \frac{1}{27^{2x}}$

b) $2^{2x} - 6(2^x) + 8 = 0$

6. Sketch $f(x) = \log_2 x$. Show at least three points. State the domain, range, equation of the asymptote and the x -intercept.

7. Sketch $f(x) = 3^x$ and $g(x) = 3^{x-2} + 1$. Show at least three points for each graph. For $f(x)$, state the domain, range, equation of the asymptote, and the y -intercept. For $g(x)$, state the equation of the asymptote and the range.

Solutions:

1a) 4 b) 2 c) 3 d) 0 e) 3 f) -2

g) -3 h) 15 i) $\frac{1}{2}$

2a) $x = \frac{1}{9}$ b) $x = 16$ c) $x = 25$ d) $x = 2$ e) $x = 3$ f) $x = 3$

g) $x = 512$ h) $x = 10$ i) $x = 4$ j) $x = \sqrt{3}$ k) $x = 5$ l) $x = -\frac{2}{3}$

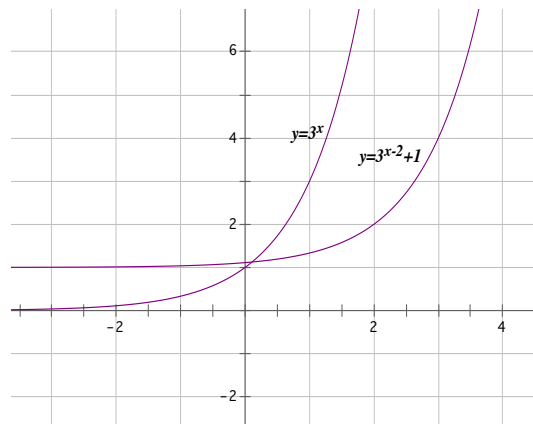
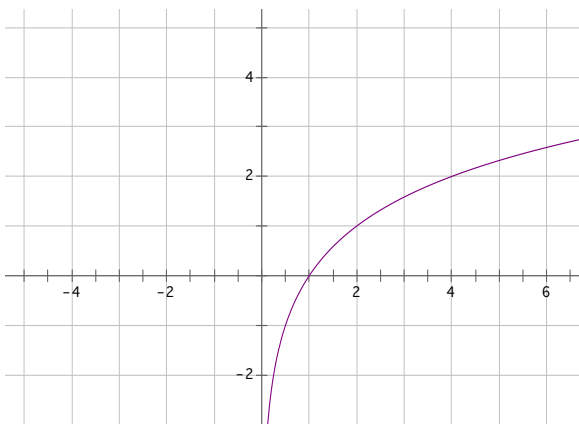
m) $x = 9$ n) $x = -2$

3a) $\frac{1}{64}$ b) 2 c) 27 d) -1 e) $\frac{1}{25}$ f) $\frac{9}{4}$

4a) $16x^8y^{12}$ b) $\frac{p^3}{27k^9}$ c) $20x^8$ d) $\frac{1}{72xy^7}$

5a) $x = \frac{1}{4}$ b) $x = 1, x = 2$

6. $x \in \mathbb{R} | x > 0, y \in \mathbb{R}, x = 0, (1,0)$ 7. $x \in \mathbb{R}, y \in \mathbb{R} | y > 0, y = 0, (0,1); y = 1, y \in \mathbb{R} | y > 1$



Review Worksheet #3**Laws of Logs - Extra Practice**

1. Solve $\log_4 x^2 - \log_4 x = 1.5$
2. Simplify $\log_2 \frac{5}{3} - \log_2 \frac{7}{6} - \log_2 \frac{5}{28}$
3. Solve $\log_5 \sqrt[3]{x} + \frac{2}{3} \log_5 x = 3$
4. Solve $\log x^3 - \log x = 4$
5. Simplify without a calculator, $\frac{\log_3 16}{\log_3 4}, \frac{\log 27}{\log 9}$
6. Solve $(\log x)^2 - \log x^2 = \log 1000$
7. Solve $(\log_4 x)^2 - \log_4 x^2 = 8$
8. Explain how you would use logarithms to solve $4^x = 3$. You may use a calculator.
9. Solve $(2.5)^x = 5$
10. Solve $\log_4(x+3) + \log_4(x-3) = 4^{\log_4 2}$
11. Solve $2 \log_2 x = 3 \log_2 10 - \log 1000$
12. Solve $\log \sqrt{x} + \log \sqrt[3]{x} = \frac{5}{3}$
13. Solve, using a calculator $2^{3x} = 5^{x+2}$
14. Solve, using a calculator $2^{x+1} = 7$
15. Solve $\log_7(x+1) + \log_7(x-5) = 1$
16. Solve $\log_6 x - \log_6(x-1) = \log_6 3$

Answers:

1. 8

2. 3

3. 125

4. 100

5. $2, \frac{3}{2}$

6. 10 or $\frac{1}{1000}$

7. 16 or $\frac{1}{256}$

8. 0.7925

9. 1.756

10. 5

11. $5\sqrt{5}$

12. 100

13. 6.849

14. 1.807

15. 6

16. $\frac{3}{2}$

Review Worksheet #4**Logarithms Practice**

1. Evaluate: $2\log_5 10 - \log_5 4$
2. Evaluate: $\log_2 56 - \log_4 49$
3. Show that $\log_{\frac{1}{2}} 5$ is equivalent to $-\log_2 5$.
4. What is the value of $25^{(-\log_5 \sqrt{2})}$?
5. Write a simplified expression for $\frac{\log x^2 + \log x^5}{\log x^5 - \log x^3}$.
6. Evaluate: $\log_3 \frac{1}{9}$
7. Write $\frac{\log 6}{\log 3}$.
8. Find x if $x = (\log_6 15)(\log_{15} 36)$.
9. Evaluate: $10^{\log_{100} 9}$
10. Solve for x : $\log_2 x = -3$
11. Solve: $\log_x 16x = 3$
12. Solve: $\log_2(x-3) + \log_2(x-1) = 3$
13. Solve for x : $2\log x + \log 3 = \log(2+x)$.
14. Solve for x to the nearest hundredth: $3^{x-1} = 2^x$
15. Solve for x : $\log_{\sqrt[3]{x}} 81 = 4$
16. Solve for x : $(\log_3 x)^2 + \log_3 x - 6 = 0$
17. Solve: $\log_2(\log_3 x) = 2$

18. Solve the following system for (x, y) :

$$\log y = \log(5^x + 2)$$

$$\log_5 y = 2x$$

19. Laurel invests \$1200 at 8% interest compounded annually. Marco invests \$900 at 11% interest compounded annually. To 1 decimal place, how long will it take before Marco has as much money as Laurel?

20. A bacteria doubles its population in 8 hours. At this rate, how many hours would it take the population of the bacteria to triple? Answer to 1 decimal place.

21. 200 grams of a radioactive substance decays to 80 grams after 300 years. To the nearest year, what is the half-life of the substance?

22. In 1 year, the world's birth rate has increased 3%. If this rate continues, then how many years (to the nearest tenth of a year) will it take to double the world's population?

23. Given the function $y - \log_3(x + 2) - 2 = 0$,

- Graph the function. Clearly show at least 2 points on the curve and state the equation of the asymptote.
- Calculate the zero of this function.

24. Solve: $8^{3x-12} = 32^{x+4}$

25. Simplify: $\frac{25^{3a+1} \times 5^{a-3}}{125^a}$.

26. Solve for x : $4^x + 4^{x-\frac{1}{2}} = 45$

27. Solve for p : $3^{p+1} + 3^{2p} = 70$

28. Solve for m : $(3^m)(5^{m+1}) = 12^{2m-1}$

29. Find all points of intersection of the graphs $y = 2\log_3(9x)$ and $y = \log_3(x + 8) + 2$. Graph both on the same set of axes to verify your answer graphically.

30. Solve for x : $\log_4 x + \log_8 \sqrt{x} = \frac{4}{3}$

Answers:

1. 2
2. 3
3. (show)
4. $\frac{1}{2}$
5. $\frac{7}{2}$
6. 2
7. $\log_3 6$
8. 2
9. 3
10. $\frac{1}{8}$
11. 4
12. 5
13. 1
14. 2.71
15. 27
16. $\frac{1}{27}, 9$
17. 81
18. $(\log_5 2, 4)$
19. 10.5 years
20. 12.7 hours
21. 227 years
22. 23.4 years
23. b) 7
24. 14
25. 5^{4a-1}
26. 2.45
27. 1.77
28. 1.81
29. (1, 4)
30. $x = 64, 4$

23a)

