INTEGRATING ALGEBRAIC EXPRESSIONS

WORKSHEET 1

(a) $\int \left(2x^9 - 3x^6 + 12x^3 - 3\right) dx$

(b) $\int \left(x^{\frac{2}{3}} - \frac{1}{x^{\frac{2}{3}}} + \frac{2}{x^5} - \frac{2}{x} \right)$

(c) $\int \sqrt[3]{x^2}\, dx$

(d) $\int \frac{1}{(3x)^2} dx$

(e) $\int \frac{x+1}{\sqrt{x}}\,dx$

(f) $\int (x+1)(3x-2)\,dx$

(g) $\int \frac{2x+1}{2x}\,dx$

(h) $\int \frac{(x-2)^2}{x^2}\,dx$

(i) $\int \frac{3x^2+x+1}{x+1}\,dx$

SOLUTIONS

(a) $\int \left(2x^9 - 3x^6 + 12x^3 - 3\right) dx = \frac{x^{10}}{5} - \frac{3x^7}{7} + 3x^4 - 3x + c$

(b) $\int \left(x^{2/3} - \frac{1}{x^{2/3}} + \frac{2}{x^5} - \frac{2}{x} \right) = \frac{3}{5} x^{5/3} - 3x^{1/3} - \frac{1}{2x^4} - 2\log_e x + c$

(c) $\int \sqrt[3]{x^2}\, dx = \int x^{2/3}\, dx = \frac{x^{5/3}}{5/3} + c$

$= \frac{3x^{5/3}}{5} + c$

(d) $\int \frac{1}{(3x)^2}\, dx = \int \frac{1}{9x^2}\, dx = \int \frac{x^{-2}}{9}\, dx$

$= -\frac{1}{9x} + c$

(e) $\int \frac{x+1}{\sqrt{x}}\, dx = \int \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\, dx = \int x^{1/2} + x^{-1/2}\, dt$

$= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + c$

$= \frac{2}{3} x^{3/2} + \frac{1}{2} x^{1/2} + c$

(f) $\int (x+1)(3x-2)\, dx = \int (3x^2 + x - 2)\, dx$

$= x^3 + \frac{x^2}{2} - 2x + c$

(g) $\int \frac{2x+1}{2x}\, dx = \int \left(\frac{2x}{2x} + \frac{1}{2x} \right) dx = \int 1 + \frac{1}{2}\left(\frac{1}{x}\right) dx$

$= x + \frac{1}{2}\log_e x + c, \; x > 0$

(h) $$\int \frac{(x-2)^2}{x^2}\,dx = \int\left(\frac{x^2-4x+4}{x^2}\right)dx = \int\left(\frac{x^2}{x^2}-\frac{4x}{x^2}+\frac{4}{x^2}\right)dx = \int\left(1-\frac{4}{x}+4x^{-2}\right)dx$$

$$= x-4\log_e x-\frac{4}{x}$$

(i) $$\int \frac{3x^2+x+1}{x+1}\,dx$$

Simplify by long division:

$$\begin{array}{r} 3x-2 \\ x+1\overline{)\,3x^2+x+1} \\ \underline{3x^2+3x} \\ -2x+1 \\ \underline{-2x-2} \\ 3 \end{array}$$

$$\int\left(\frac{3x^2+x+1}{x+1}\right)dx = \int\left(3x-2+\frac{3}{x+1}\right)dx$$

$$= \frac{3x^2}{2}-2x+3\log_e(x+1)+c,\ x>-1$$