CALCULUS BC
WORKSHEET 2 ON PARTICLE MOTION

Work these on **notebook paper**. Use your calculator, and give decimal answers correct to three decimal places. Write your justifications in a sentence.

1. A particle moves along a horizontal line so that its position at any time $t \geq 0$ is given by $s(t) = -t^3 + 7t^2 - 14t + 8$, where s is measured in meters and t in seconds.

(a) Find the instantaneous velocity at any time t and when $t = 2$.
(b) Find the acceleration of the particle at any time t and when $t = 2$.
(c) When is the particle at rest? When is moving to the right? To the left? Justify your answers.
(d) Find the displacement of the particle during the first two seconds?
(e) Find the total distance traveled by the particle during the first two seconds?
(f) Are the answers to (d) and (e) the same? Explain.
(g) When is the particle speeding up? Slowing down? Justify your answers.

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2. The position of a particle at time t seconds, $t \geq 0$, is given by $s(t) = t^2 - \sin t,\ 0 \leq t \leq 3$, where t is measured in seconds and s is measured in meters. Find the particle's acceleration each time the velocity is zero.

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3. A particle's velocity at time t seconds, $t \geq 0$, is given by $v(t) = \cos\left(t^2\right) + t,\ 0 \leq t \leq 2$, where t is measured in seconds and v is measured in meters/second. Find the velocity of the particle each time the acceleration is zero.

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4. A particle moves along the x-axis so that at any time $t > 0$, its velocity is given by $v(t) = \ln\left(1 + 2^t\right)$. Find the acceleration of the particle when $t = 3$.

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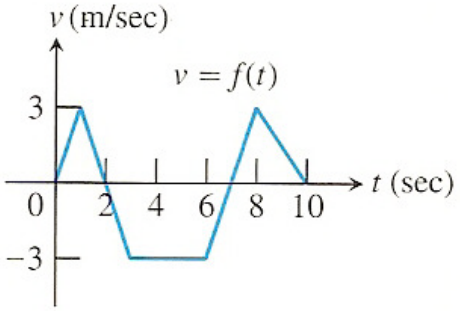
5. The figure on the right shows the velocity of a particle moving along a vertical line.

(a) When is the particle moving up? moving down? at rest? Justify your answers.
(b) When is the particle moving at a constant speed?
(c) Graph the particle's speed for $0 \leq t \leq 10$.
(d) Graph the particle's acceleration for $0 < t < 10$.



TURN->>>

6. (2004) A particle moves along the y-axis so that its velocity at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$.

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

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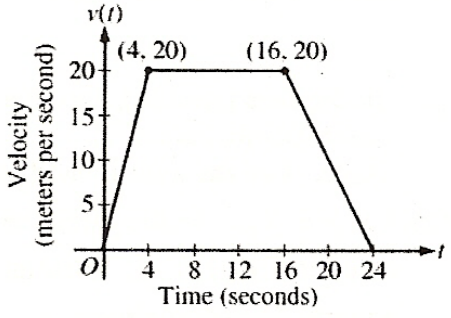
7. (2005) A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph on the right.



(a) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(b) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(c) Find the average rate of change of v over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

Answers

1. (a) $-3t^2+14t=14,\ \ 2\,\text{m/sec}$

   (b) $-6t+14,\ 2\,\text{m/sec}^2$

   (c) At rest at $t = 1.451$ and $t = 3.215$ because $v(t)=0$ there. Moving left for [0, 1.451) and $(3.215, \infty)$ because $v(t)<0$. Moving right for (1.451, 3,215) because $v(t)>0$.

   (d) – 8 m

   (e) 9.262 m

   (f) No, the displacement and distance are not the same because the particle changed direction at $t = 1.451$.

   (g) Slowing down on (0, 1.451) and (2.333, 3.215) because vel. and acc. have opposite signs. Speeding up on (1.451, 2.333) and $(3.215, \infty)$ because vel. and acc. have the same sign.

2. $a(0.45018...)=2.435\,\text{m/sec}^2$

3. $v(0.81305\ldots) = 1.602\ \text{m/s}$ $\qquad$ $v(1.6853\ldots) = 0.730\ \text{m/s}$

4. 0.616

5. (a) Moving up for (0, 2) and (7, 10) because $v(t)>0$. Moving down for (2, 7) because $v(t)<0$. At rest at $t = 2$ and at $t = 7$ because $v(t)=0$ there.

   (b) (3, 6)

   (c) and (d) Graphs

6. (a) – 0.133

   (b) – 0.436. Speed is increasing at $t = 2$ because $v(t)$ and $a(t)$ are both negative.

   (c) $v(t)=0$ when $t = 0.443$. This is the only critical number. $v(t)>0$ for (0, 0.443) and $v(t)<0$ for $(0.443, \infty)$ so the particle reaches its highest point at $t = 0.443$.

7. (a) $v'(4)$ does not exist because the graph of $v(t)$ has a sharp turn at $t = 4$.

   $v'(20)=-\frac{5}{2}\,\text{m/sec}^2$.

   (b) $a(t)=\begin{cases}5,\ 0<t<4\\ 0,\ 4<t<16\\ -\frac{5}{2},\ 16<t<24\end{cases}$

   (c) Ave. rate of change = $-\frac{5}{6}\,\text{m/sec}^2$. No, the MVT does not apply for $8 < c < 20$ because the graph of $v(t)$ is not differentiable at $t = 16$.