Cheat Sheet: Techniques in Differentiation – 1

Basic Rules

$$\frac{d}{dx}\left(ax^n\right) = anx^{n-1},\ n \neq 0$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(k) = 0$$

$$\frac{d}{dx}\left(ax^{-n}\right) = -anx^{-n-1} = -\frac{an}{x^{n+1}},\ n \neq 0$$

$$\frac{d}{dx}\left(f(x)\right) = f'(x)$$

$$\frac{d}{dx}\left(af(x)\right) = a\frac{d}{dx}\left(f(x)\right) = af'(x)$$

$$\frac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) \pm g'(x)$$

$$\frac{d}{dx}\left(\sin x\right) = \cos x$$

$$\frac{d}{dx}\left(\cos x\right) = -\sin x$$

$$\frac{d}{dx}\left(\tan x\right) = \sec^2 x$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}\left(\log_e x\right) = \frac{1}{x},\ x > 0$$

DO NOT lower powers on trigonometric, logarithmic or exponential expressions.

Definitions

Derivative:

The gradient of a tangent to a curve

Notations:

$$y',\ \frac{d}{dx},\ \frac{dy}{dx},\ \frac{d[f(x)]}{dx},\ f'(x),\ D_x[y],$$

$$D_x[f(x)],\ \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Given $y = f(u)$ where u is a function of x, then: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Note: $u =$ the **inside** function

$u =$ bracketed terms
$u =$ power in exponentials
$u =$ base numeral in logarithms
$u =$ angle in trigonometric functions

Example: $\frac{d}{dx}\left(5(1-x^2)^4\right)$

Let $u = 1 - x^2$ $\quad \therefore \frac{du}{dx} = -2x$

Let $y = 5u^4$ $\quad \therefore \frac{dy}{du} = 20u^3$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 20u^3 \times -2x = -40xu^3$$

$$= -40x(1-x^2)^3$$

Example: $\frac{d}{dx}\left(-3\log_e(\tan x)\right)$

Let $u = \tan x$ $\quad \therefore \frac{du}{dx} = \sec^2 x$

Let $y = -3\log_e u$ $\quad \therefore \frac{dy}{du} = -\frac{3}{u}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= -\frac{3}{u}.\sec^2 x = -\frac{3\sec^2 x}{\tan x}$$

Quick Chain Rule

Brackets with high, fractional or negative powers:

$$\frac{d}{dx}\left[a\left(f(x)\right)^n\right] = anf'(x)\left(f(x)\right)^{n-1}$$

$\frac{dy}{dx} =$ Power × Derivative of contents of bracket × Given expression

but lower the power on the brackets by one

Example: $\frac{d}{dx}\left(5(1-x^2)^4\right) = 4 \times -2x \times 5(1-x^2)^3 = -40x(1-x^2)^3$

Quick Chain Rule

Exponential functions (base e only)

$$\frac{d}{dx}\left(ae^{f(x)}\right) = af'(x)e^{f(x)}$$

$\frac{dy}{dx} =$ Derivative of the power × Given term

Example: $\frac{d}{dx}\left(-\frac{3e^{\sin x}}{2}\right) = \cos x \times -\frac{3e^{\sin x}}{2} = -\frac{3e^{\sin x}.\cos x}{2}$

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Cheat Sheet: Techniques in Differentiation – 2

Quick Chain Rule

Logarithmic functions (base e only)

$$\frac{d}{dx}\left(a\log_e f(x)\right)=\frac{af'(x)}{f(x)},\ f(x)>0$$

$$\frac{dy}{dx}=Coefficient\ of\ log_e\times\frac{Derivative\ of\ base\ numeral}{Base\ numeral}$$

Example: $\frac{d}{dx}\left(-3\log_e(\tan x)\right)=-\frac{3\sec^2 x}{\tan x}$

Method: Finding Derivatives

Step 1: Rewrite all terms as powers on x.

eg. $\sqrt[3]{x}=x^{\frac{1}{3}}$

Step 2: Bring terms involving x in the denominator to the top by changing the sign on the power.

eg. $\frac{3}{2x^4}=\frac{3x^{-4}}{2}$

Step 3: Simplify expressions.
Step 4: Differentiate.
Step 5: Re-write using positive powers.

Quick Chain Rule

Trigonometric functions

$$\frac{d}{dx}\left[a\sin\left(f(x)\right)\right]=af'(x)\cos\left(f(x)\right)$$

$$\frac{d}{dx}\left[a\cos\left(f(x)\right)\right]=-af'(x)\sin\left(f(x)\right)$$

$$\frac{d}{dx}\left[a\tan\left(f(x)\right)\right]=af'(x)\sec^2\left(f(x)\right)$$

i.e. $\frac{dy}{dx}=$ Coefficient of trig $\times$ Derivative of the angle $\times$ Derived trigonometric function with the original angle

Example: $\frac{d}{dx}\left(\frac{1}{3}\cos(x^3+2)\right)=\frac{1}{3}\times 3x^2\times -\sin(x^3+2)=-x^2\sin(x^3+2)$

Product Rule

The Product Rule states that given two functions (denoted as u and v), then:

$$\frac{d}{dx}(u.v)=u.\frac{dv}{dx}+v.\frac{du}{dx}=(First\times Do\ Second)+(Second\times Do\ First)$$

Example: $\frac{d}{dx}(\sin x.\cos x)=(\sin x\times -\sin x)+(\cos x\times\cos x)=\sin^2 x+\cos^2 x=1$

Specific Points

To find the derivative at a specific point substitute the value of x into the derivative expression.

Example: Derivative at $x=3$ is $f'(3)$.

Quotient Rule

The Quotient Rule states that given $y=\frac{u}{v}, v\neq 0$, then:

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v.\frac{du}{dx}-u.\frac{dv}{dx}}{v^2}=\frac{(Bottom\times Do\ Top)-(Top\times Do\ Bottom)}{Bottom^2}$$

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Example: $\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)=\frac{(\cos x\times\cos x)-(\sin x\times -\sin x)}{\cos^2 x}=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}=\frac{1}{(\cos x)^2}=\sec^2 x$