UNIT 3 & 4 MATHEMATICAL METHODS

INTEGRATION TECHNIQUES

MULTIPLE CHOICE QUESTIONS

Instructions for Section 1

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers. You should attempt every question.

No marks will be given if more than one answer is completed for any question.

QUESTION 1

If $\int \left(ae^{bx}\right) dx = -2e^{2x} + c$ then

A. $a = 4,\ b = -2$

B. $a = -2,\ b = 2$

C. $a = -1,\ b = 2$

D. $a = -4,\ b = 2$

E. $a = -4,\ b = -2$

QUESTION 2

$\int \left(\frac{-3a}{2ax+b}\right) dx =$

A. $\log_e(2ax+b)+c$

B. $-\frac{3}{2}\log_e(2ax+b)+c$

C. $\frac{3}{2}\log_e(2ax+b)+c$

D. $-3\log_e(2ax+b)+c$

E. $3\log_e(2ax+b)+c$

QUESTION 3

Given that the derivative of $x\log_e(3x)$ is equal to $1+\log_e(3x)$, then $\int 2\log_e(3x)\,dx$ is equal to

A. $x\log_e(3x)-c$

B. $2x\log_e(3x-1)-c$

C. $2x\log_e(3x)-c$

D. $2x(\log_e(3x)-1)-c$

E. $x\log_e(3x)-1-c$

QUESTION 4

Given that $\frac{dy}{dx}=ae^x+1$, and when $x=1$, $\frac{dy}{dx}=3$ and $y=2$, the value of y when $x=0$ is

A. $y=1$

B. $y=1-\frac{2}{e}$

C. $y=\frac{2}{e}-1$

D. $y=\frac{4}{e}-1$

E. $y=\frac{2}{e}+1-2e$

QUESTION 5

If $g(x)=f(\cos(x))$ and $g'(x)=-3\cos^2(x)\sin(x)$ then $f(x)$ is equal to

A. $-3x^2+c$

B. $3x^2+c$

C. $-x^3+c$

D. x^3+c

E. $-\cos^3 x$

QUESTION 6

If $\frac{d}{dx}\left(g(5x)\right)=g'(x)$ and $g'(1)=10$ and $g(1)=0$ then $g'(5)$ equals

A. 0

B. 1

C. 2

D. 5

E. 10

QUESTION 7

If $y=F(x)$ and $\frac{dy}{dx}=f(x)$ then $\int_1^2 f(x)\,dx$ is equal to

A. $F(2)-F(1)$

B. $f(2)-f(1)$

C. $f(2)-F(1)$

D. $F(2)-f(1)$

E. $f(x)+c$

QUESTION 8

If $\int_a^2 (2x)\,dx=0$ then the value of a is

A. ± 2

B. 0

C. -2

D. ± 4

E. -4

QUESTION 9

If $\int_0^a \left(\frac{2}{4x+1}\right)dx=\log_e k$ then k is equal to

A. $\sqrt{4a+1}$

B. $(4a+1)^2$

C. $4a+1$

D. $e^{\frac{8}{(4a+1)^2}}-8$

E. $e^{\frac{2}{(4a+1)^2}}-2$

SOLUTIONS

QUESTION 1 Answer is D

$$\int\left(ae^{bx}\right)dx=\frac{a}{b}e^{bx}+c$$

Equate cofficients: $\frac{a}{b}e^{bx}+c=-2e^{2x}+c$

Therefore: $b=2$ and $\frac{a}{2}=-2 \quad \therefore a=-4$

QUESTION 2 Answer is B

QUESTION 3 Answer is D

$$\frac{d}{dx}\left(x\log_e(3x)\right)=1+\log_e(3x)$$

Integrate both sides:

$$x\log_e(3x)=x+c_1+\int\log_e(3x)dx$$

$$\therefore\int\log_e(3x)dx=x\log_e(3x)-x-c_1$$

$$\therefore\int 2\log_e(3x)dx=2(x\log_e(3x)-x-c_2)$$

$$=2x(\log_e(3x)-1)-c$$

QUESTION 4 Answer is C

When $x=1$, $\frac{dy}{dx}=3$: $ae^x+1=3$

$$a=\frac{2}{e}$$

$$y=\int(ae^x+1)\,dx=ae^x+x+c$$

When $x=1$, $y=2$: $ae+1+c=2$

$$ae+c=1$$

Substitute $a=\frac{2}{e}$: $\frac{2}{e}e+c=1 \qquad \therefore c=-1$

$$y=ae^x+x+c=\frac{2}{e}e^x+x-1$$

$$y=2e^{x-1}+x-1$$

When $x=0$: $y=2e^{-1}-1=\frac{2}{e}-1$

QUESTION 5 Answer is D

$$g(x) = f(\cos(x))$$
$$g'(x) = -f'(\cos x)\sin x$$

Given: $g'(x) = -3\cos^2(x)\sin(x)$

Equating gives: $f'(\cos x) = 3\cos^2 x = 3(\cos x)^2$

$$f'(x) = 3x^2$$
$$f(x) = \int (3x^2)\, dx = x^3 + c$$

QUESTION 6 Answer is C

Let $y = g(5x)$ and $u = 5x$

$$\therefore \frac{dy}{dx} = 5g'(5x)$$

As $\frac{dy}{dx} = \frac{d}{dx}(y)$ and $y = g(5x)$:

$$\frac{dy}{dx} = \frac{d}{dx}(y) = 5g'(5x)$$

$$\frac{d}{dx}(g(5x)) = 5g'(5x)$$
$$\therefore g'(x) = 5g'(5x)$$

When $x = 1$: $g'(1) = 5g'(5)$

As $g'(1) = 10$ then $10 = 5g'(5)$ $\therefore g'(5) = 2$

QUESTION 7 Answer is A

$$\int_1^2 f(x)\, dx = \left[F(x)\right]_1^2 = F(2) - F(1)$$

QUESTION 8 Answer is A

QUESTION 9 Answer is A

$$\int_0^a \left(\frac{2}{4x+1}\right) dx = \frac{1}{2}\int_0^a \left(\frac{4}{4x+1}\right) dx = \frac{1}{2}\left[\log_e(4x+1)\right]_0^a = \frac{1}{2}\left[\log_e(4a+1) - \log_e 1\right]$$

$$= \frac{1}{2}\log_e(4a+1) = \log_e(4a+1)^{1/2}$$

$$\log_e(4a+1)^{1/2} = \log_e k$$

$$k = \sqrt{4a+1}$$