Examples of Solving Logarithmic Equations

Steps for Solving Logarithmic Equations Containing Only Logarithms

Step 1: Determine if the problem contains only logarithms. If so, go to Step 2. If not, stop and use the Steps for Solving Logarithmic Equations Containing Terms without Logarithms.

Step 2: Use the properties of logarithms to simplify the problem if needed. If the problem has more than one logarithm on either side of the equal sign then the problem can be simplified.

Step 3: Rewrite the problem without the logarithms.

Step 4: Simplify the problem.

Step 5: Solve for x.

Step 6: Check your answer(s). Remember we cannot take the logarithm of a negative number, so we need to make sure that when we plug our answer(s) back into the original equation we get a positive number. Otherwise, we must drop that answer(s).

Steps for Solving Logarithmic Equations Containing Terms without Logarithms

Step 1: Determine if the problem contains only logarithms. If so, stop and use Steps for Solving Logarithmic Equations Containing Only Logarithms. If not, go to Step 2.

Step 2: Use the properties of logarithms to simplify the problem if needed. If the problem has more than one logarithm on either side of the equal sign then the problem can be simplified.

Step 3: Rewrite the problem in exponential form.

Step 4: Simplify the problem.

Step 5: Solve for x.

Step 6: Check your answer(s). Remember we cannot take the logarithm of a negative number, so we need to make sure that when we plug our answer(s) back into the original equation we get a positive number. Otherwise, we must drop that answer(s).

Example – Solve: $\log_4(4x-9)=3$

$\log_4(4x-9)=3$	This problem contains terms without logarithms.
$\log_4(4x-9)=3$	This problem does not need to be simplified because there is only one logarithm in the problem.
$4x-9=4^3$	Rewrite the problem in exponential form by moving the base of the logarithm to the other side.
$4x-9=64$	Simplify the problem by cubing the 4.
$x=\frac{73}{4}$	Solve for x by adding 9 to each side and then dividing each side by 4.
$x=\frac{73}{4}$	Check the answer; this is an acceptable answer because we get a positive number when it is plugged back in.

Therefore, the solution to the problem $\log_4(4x-9)=3$ is $x=\frac{73}{4}$.

Example – Solve: $\log_9(3x+5)=\log_9(7x-12)$

$\log_9(3x+5)=\log_9(7x-12)$	This problem contains only logarithms.
$\log_9(3x+5)=\log_9(7x-12)$	This problem does not need to be simplified because there is only one logarithm on each side of the problem.
$3x+5=7x-12$	Drop the logarithms.
$3x+5=7x-12$	This problem does not need to be simplified.
$x=\frac{17}{4}$	Solve the problem by subtracting 7x from each side, subtracting 5 from each side, and finally dividing by –4.
$x=\frac{17}{4}$	Check the answer; this is an acceptable answer because we get a positive number when it is plugged back in.

Therefore, the solution to the problem $\log_9(3x+5)=\log_9(7x-12)$ is $x=\frac{17}{4}$.

Example – Solve: $\ln(3x+11)=4$

$\ln(3x+11)=4$	This problem contains terms without logarithms.
$\ln(3x+11)=4$	This problem does not need to be simplified because there is only one logarithm in the problem.
$3x+11=e^4$	Rewrite the problem in exponential form by moving the base of the logarithm to the other side. For natural logarithms the base is e.
$3x+11\approx 54.598150$	Simplify the problem by raising e to the fourth power. Round the answer as appropriate, these answers will use 6 decimal places.
$x\approx 14.532717$	Solve for x by subtracting 11 from each side and then dividing each side by 3.
$x\approx 14.532717$	Check the answer; this is an acceptable answer because we get a positive number when it is plugged back in.

Therefore, the solution to the problem $\ln(3x+11)=4$ is $x\approx 14.532717$.

Example – Solve: $\log_6 x+\log_6(x+5)=2$

$\log_6 x+\log_6(x+5)=2$	This problem contains terms without logarithms.
$\log_6(x(x+5))=2$	This problem can be simplified by using Property 3 which changes the addition of logarithms to multiplication.
$x(x+5)=6^2$	Rewrite the problem in exponential form by moving the base of the logarithm to the other side.
$x^2+5x=36$	Simplify the problem by distributing and squaring the 6.
$x^2+5x-36=0$ $(x-4)(x+9)=0$	Solve the problem by subtracting 36 from each side to get it equal to zero, and then factoring or using the quadratic formula to find the values of x.
$x=4$ or $x=-9$	
$x=4$	Check the answers, only one answer is acceptable because the other answer produces a negative number when it is plugged back in.

Therefore, the solution to the problem $\log_6 x+\log_6(x+5)=2$ is $x=4$.

Example – Solve: $\log_7(2x-3)-\log_7(x+2)=1$

$\log_7(2x-3)-\log_7(x+2)=1$	This problem contains only logarithms.
$\log_7\left(\frac{2x-3}{x+2}\right)=1$	This problem can be simplified by using Property 4 which changes the subtraction of logarithms to division.
$\left(\frac{2x-3}{x+2}\right)=7^1$	Rewrite the problem in exponential form by moving the base of the logarithm to the other side.
$\left(\frac{2x-3}{x+2}\right)=7$	Simplify the problem by raising 7 to the first power.
$2x-3=7(x+2)$ $2x-3=7x+14$ $x=-\frac{17}{5}$	Solve for x by cross-multiplying, distributing, subtracting 7x from each side, adding 3 to each side, and finally dividing each side by –5.
No Solution	Check the answers, this problem has "No Solution" because the only answer produces a negative number and we can't take the logarithm of a negative number.

Therefore, the problem $\log_7(2x-3)-\log_7(x+2)=1$ has no solution.

Example – Solve: $\ln(2x-1)+\ln(x+3)=\ln(x^2-3x)$

$\ln(2x-1)+\ln(x+3)=\ln(x^2-3x)$	This problem contains only logarithms.
$\ln((2x-1)(x+3))=\ln(x^2-3x)$	This problem can be simplified by using Property 3 which changes the addition of logarithms to multiplication.
$(2x-1)(x+3)=x^2-3x$	Drop the logarithms.
$2x^2-5x-3=x^2-3x$	Simplify the problem by distributing or FOILing and combining like terms.
$x^2-2x-3=0$ $(x+1)(x-3)=0$ $x=-1$ or $x=3$	Solve the problem by subtracting x^2 from each side and adding 3x to each side to get it equal to zero, and then factoring or using the quadratic formula to find the values of x.
$x=3$	Check the answers, only one answer is acceptable because the other answer produces a negative number when it is plugged back in.

Therefore, the solution to the problem $\ln(2x-1)+\ln(x+3)=\ln(x^2-3x)$ is $x=3$.