SOLVING WORD PROBLEMS

Word problems can be classified into different categories. Understanding each category will give be an advantage when trying to solve word problems. All problems in each category are solved the same way. With practice you will begin to see the similarities among word problems and be able to solve them easier and faster.

Age Problems

These types of problems ask you to figure out the age of different people by giving you different clues.

Example: John is 3 years older than Jim. Jim is 4 years less than twice David's age. How old are the three boys if their ages add up to 35?

Let: David's age = x ← Translate words into mathematical expressions that represent each boy's age.

Jim's age = $2x-4$

John's age = $\underbrace{(2x-4)}_{\text{Jim's age}}+3$

$x+2x-4+(2x-4)+3=35$ ← Ages of the three boys together equal 35.

$5x=40$ ← Solve for x

$x=8$ ← David's age

$2x-4=12$ ← Jim's age

$(2x-4)+3=15$ ← John's age

Coin Problems

These types of problems involve figuring out the number of coins per given value.

Example: A cashier has 3 more dimes than nickels and twice as many nickels as quarters. Find the number of each kind of coin if the total value of the coins is $3.05.

Let: # of quarters = x ← Translate words into mathematical expressions representing the number of coins for each value.

\# of nickels = $2x$

\# of dimes = $2x+3$

$0.25x+0.05(2x)+0.10(2x+3)=3.05$ ← Multiply the number of coins times their face value.

$0.25x+0.10x+0.20x+0.30=3.05$

$0.55x=2.75$

$x=5$ ← Total # of quarters

$2x=10$ ← Total # of nickels

$2x+3=13$ ← Total # of dimes

Distance Problems

Example: You are driving along at 55 mph when you are passed by a car doing 85 mph. How long will it take for the car that passed you to be one mile ahead of you?

Let D be the distance that you travel in time t, and $D + 1$ be your distance plus one mile ahead of you that the other car traveled in time t.

Using the rate equation in the form *distance = speed • time,* or $D = s \cdot t$ for each car, we can write:

$D = 55\,t$ and $D + 1 = 85\,t$

Substituting the first equation into the second → $55t + 1 = 85t$

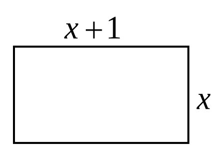
and solving for t → $-30t = -1$

$t = 1/30$ hours or **2 minutes**

Geometry Problems

Example: If the perimeter of a rectangle is 18 inches, and one side is one inch longer than the other, how long are the sides?

Let one side be x (width) and the other side be $x + 1$ (length).
The perimeter of a rectangle is found by the formula: $P = 2w + 2l$
Then the given condition may be expressed as:
$2x + 2(x + 1) = 18$



$2x + 2x + 2 = 18$
$4x + 2 = 18$
$x = 4$ Therefore, the sides have length 4 and 5.

Investment Problems

Example: Suppose $10,000 is invested at 9% interest. How much money must be invested at 12% to produce a return of 11% on the entire amount invested?

Let: amount invested at 12% = x
amount invested at 9% = 10,000
amount invested at 11% = x + 10, 000

$0.12x + 0.09(10{,}000) = 0.11(x + 10{,}000)$
$12x + 9(10{,}000) = 11(x + 10{,}000)$ ⟶ after multiplying both sides by '10'.
$12x + 90{,}000 = 11x + 110{,}000$
$x = \$20{,}000$

Mixture Problems

Example: The instructions on a can of powdered drink mix say to mix 1/4 cup of the mix with 2 quarts of water. How much of the mix should be used with 1 1/2 gallons of water?

Let x = # of cups of the drink mix to use

$$\frac{1/4}{2} = \frac{x}{6}$$ ← There are 6 quarts in 1 ½ gallons

$$6\left(\frac{1}{4}\right) = 2x$$

$$\frac{3}{2} = 2x$$

$$x = \frac{3}{4} \text{ cup of the drink mix}$$

Number Problems

Example: Find a number such that 5 more than one-half the number is three times the number.

Translating into math: $5 + \frac{x}{2} = 3x$ ← let x be the unknown number.

Solving: $2\left(5 + \frac{x}{2}\right) = 2(3x)$ →

$$10 + x = 6x$$
$$10 = 5x$$
$$x = 2$$

Percent of Problems

Example: The price of gasoline increased by 25% between January and March. If the price per gallon in March was \$1.15, what was the price per gallon in January?

To find the price in March: Price in January + 25% increase in cost = Price in March

Let: price per gallon in January = x
25% increase in cost = $0.25x$
price in March = 1.15

Then $x + 0.25x = 1.15$

$$1.25x = 1.15$$
$$x = \$0.92$$

Work Problems

Example: A fast employee can assemble 7 radios in an hour, and another slower employee can only assemble 5 radios per hour. If both employees work together, how long will it take to assemble 26 radios?

The two together will build 7 + 5 = 12 radios in an hour, so their combined rate is 12 radios per hour.

Using $\text{Time} = \dfrac{\text{Quantity}}{\text{Rate}} = \dfrac{26}{12} = 2\dfrac{1}{6}$ hour $\rightarrow$ 2 hours 10 minutes

Since many of the word problems can be solved in a similar way, the following lists a series of recommended steps to follow when solving word problems which will make the problem easier to solve.

List of Steps to Follow When Solving Word Problems

1. Read the problem carefully to determine what the problem is saying and what it is asking for. Read it as many times as necessary to understand it.

2. Read the problem again and write down details.

   A. Determine what question is being asked.

      i. **Key words –** to determine what unknown quantity you are asked to find, look for key words such as *how many*, *how much*, *what is*, *find*, *how long*.

      ii. **Associated words –** find the word or words associated with the key words.

      Examples: (Key words are underlined, associated words are in *italics*)

      How many *papers* did he sell?
      How much *money* was left?
      What are the *lengths* of the two bars?
      Find the *dimensions* of the rectangle.
      How long will it take for John to *save $200*?
      Determine the *percentage increase* in the price per unit.

      iii. For complicated problems, it may help to write down in your own words what question is being asked.

B. Write down essential details.

   i. Make diagrams, charts, or drawings to help you visualize the problem.

   ii. Assign a letter to represent the unknown quantity (or one of the unknowns if there is more than one) and write down the letter and what it represents.

   Example: Let x = cost per unit

   iii. Determine what units the solution should have and write this down.

   iv. If there is more than one unknown quantity to find, establish and write down the relationships among the unknowns.

   Example: Let x = Pat's age, then $2x - 3$ = Laura's age

   If there is no such relationship, assign other letter(s) to represent the other unknown(s). Remember that to find a unique solution you must be able to write down as many equations as you have letters representing unknowns.

   v. If there are several known quantities given in the problem, it may be helpful to write them down in tabular form.

   Example:

Amount of loan	–	\$10,000
Interest rate	–	9%
Monthly payments	–	\$200 per month

3. If still unclear as to what is being asked, or can't find the relationships among the data given, read the problem again. In some word problems, relationships are straightforward and easy to find. Other problems may require several readings before the relationships become clear. In the latter case, a chain of deductive reasoning may be required to link the data given in the problem to the problem solution. Do not get discouraged. Usually with each reading you will understand the problem a little better.

4. Write down the equation or inequality which expresses the relationships among the unknowns and all other relevant quantities in the problem.

5. Solve the equation or inequality. Identify the unknowns in the problem using the solution you obtained.

6. Check your results.