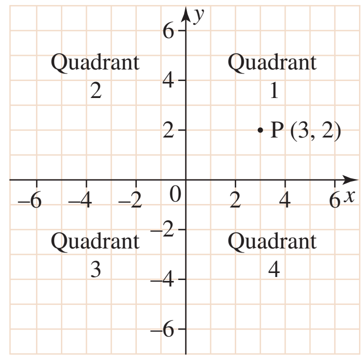
7.2 Plotting linear graphs



- The **Cartesian plane** is divided into 4 regions (quadrants) by the x- and y-axes, as shown at right.
- Every point in the plane is described exactly by a pair of coordinates (x, y). The point P (3, 2) is marked on the diagram.

Plotting from a rule

- A graph can be drawn by plotting a series of points on a Cartesian plane. To do this requires:
  1. a set of x-values
  2. a rule.

WORKED EXAMPLE 1

Plot the graph specified by the rule $y = x + 2$ for the x-values −3, −2, −1, 0, 1, 2, 3.

THINK / **WRITE/DRAW**

1 Draw a table and write in the required x-values.

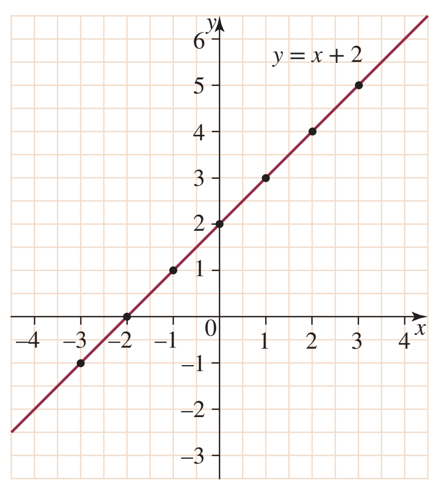
x	−3	−2	−1	0	1	2	3
y							

2 Substitute each x-value into the rule $y = x + 2$ to obtain the corresponding y-value.
When $x = -3$, $y = -3 + 2 = -1$.
When $x = -2$, $y = -2 + 2 = 0$ etc.
Write the y-values into the table.

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5

3 Plot the points from the table: (−3, 1) etc.

4 Join the points with a straight line and label the graph with its equation, $y = x + 2$.



- A straight line graph is called a **linear graph** and its rule is called a linear relation. The rule for a linear graph can always be written in the form $y = mx + c$, for example $y = 4x - 5$ or $y = x + 1.2$.

- If a graph is linear, then a minimum of two points need be plotted to locate the straight line. It is sensible to choose points that are some distance apart and to use a third point to check an error has not been made.

WORKED EXAMPLE 2

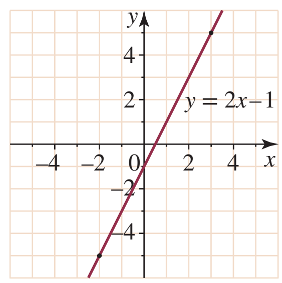
Plot two points and hence draw the linear graph $y = 2x - 1$.

THINK / **WRITE/DRAW**

1. Choose any two x-values, for example $x = -2$ and $x = 3$.
2. Calculate y by substituting each x-value into $y = 2x - 1$.
   $y = 2 \times -2 - 1 = -5$
   $y = 2 \times 3 - 1 = 5$

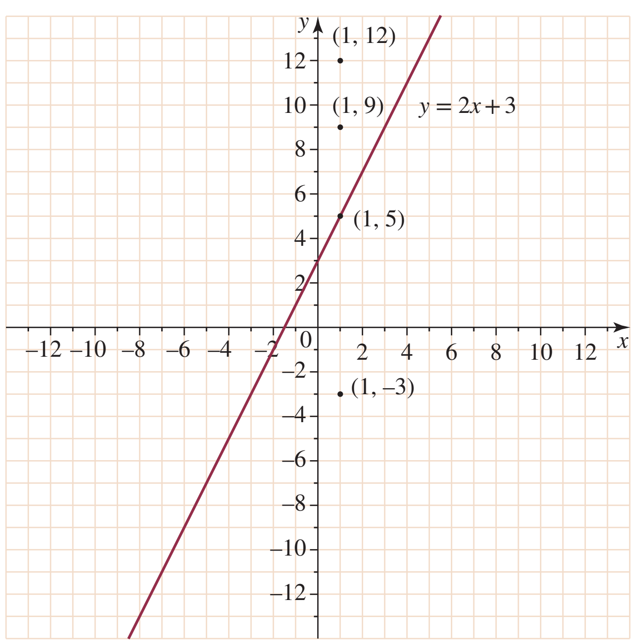
x	-2	3
y	-5	5

3. Plot the points $(-2, -5)$ and $(3, 5)$.
4. Draw a line through the points and add a label.



Points on a line

- Consider the line that has the rule $y = 2x + 3$ as shown in the graph.
  If $x = 1$, then $y = 2(1) + 3$
  $= 5$
  So the point $(1, 5)$ lies on the line $y = 2x + 3$.
- The points $(1, 0)$, $(1, -3)$, $(1, 9)$, $(1, 12)$ … are not on the line, but lie above or below it.



WORKED EXAMPLE 3

Does the point (2, 4) lie on the line given by:
a $y = 3x - 2$? **b** $x + y = 5$?

THINK	WRITE
a 1 Substitute $x = 2$ into the equation $y = 3x - 2$ and find y.	**a** $y = 3x - 2$ $x = 2$: $y = 3(2) - 2$ $= 6 - 2$ $= 4$
2 When $x = 2$, $y = 4$, so the point (2, 4) lies on the line. Write the answer.	The point (2, 4) lies on the line $y = 3x - 2$.
b 1 Substitute $x = 2$ into the equation $x + y = 5$ and find y.	**b** $x + y = 5$ $x = 2$: $2 + y = 5$ $y = 3$
2 The point (2, 3) lies on the line, but the point (2, 4) does not. Write the answer.	The point (2, 4) does not lie on the line $x + y = 5$.

assess on

Exercise 7.2 Plotting linear graphs

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 12	Questions: 1–10, 12, 13	Questions: 1–14

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REFLECTION
In linear equations, what does the coefficient of x determine?

eBook plus

Interactivity
Drawing a graph
int-1020

Digital docs
SkillSHEET
Plotting coordinate points
doc-6161
SkillSHEET
Substituting into a rule
doc-6162
SkillSHEET
Completing a table of values
doc-6163
SkillSHEET
Plotting a line from a table of values
doc-6164

FLUENCY

1 **MC** **a** The point with coordinates $(-2, 3)$ is:
- **A** in quadrant 1
- **B** in quadrant 2
- **C** in quadrant 3
- **D** in quadrant 4

b The point with coordinates $(-1, -5)$ is:
- **A** in the first quadrant
- **B** in the second quadrant
- **C** in the third quadrant
- **D** in the fourth quadrant

c The point with coordinates $(0, -2)$ is:
- **A** in the third quadrant
- **B** in the fourth quadrant
- **C** on the x-axis
- **D** on the y-axis

2 **WE1** For each of the following rules, complete the table below and plot the linear graph.

x	-3	-2	-1	0	1	2	3
y							

a $y = x$
b $y = 2x + 2$
c $y = 3x - 1$
d $y = -2x$

3 WE2 By first plotting 2 points, draw the linear graph given by each of the following.

a $y = -x$
b $y = \frac{1}{2}x + 4$
c $y = -2x + 3$
d $y = x - 3$

4 WE3 Do these points lie on the graph of $y = 2x - 5$?

a (3, 1)
b (−1, 3)
c (0, 5)
d (5, 5)

5 Does the given point lie on the given line?

a $y = -x - 7$, (1, −8)
b $y = 3x + 5$, (0, 5)
c $y = x + 6$, (−1, 5)
d $y = 5 - x$, (8, 3)
e $y = -2x + 11$, (5, −1)
f $y = x - 4$, (−4, 0)
g $y = 7x - 11$, (1, −4)
h $2x + y = 10$, (3, 4)

6 MC The line that passes through the point (2, −1) is:

A $y = -2x + 5$
B $y = 2x - 1$
C $y = -2x + 1$
D $x + y = 1$

7 Match each point with a line passing through that point.

a (1, 1)
b (1, 3)
c (1, 6)
d (1, −4)

A $x + y = 4$
B $2x - y = 1$
C $y = 3x - 7$
D $y = 7 - x$

REASONING

8 The line through (1, 3) and (0, 4) passes through every quadrant except one. Which one? Explain your answer.

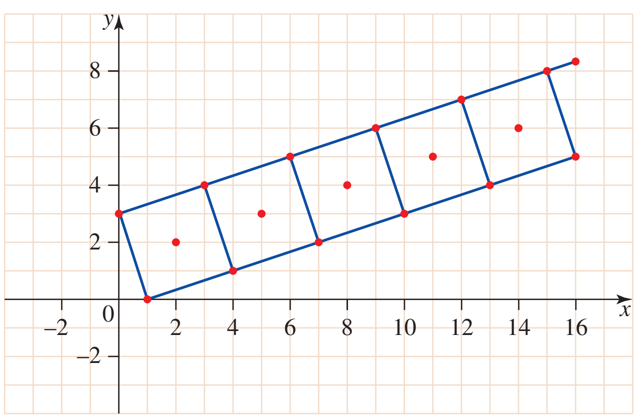
9 a Which quadrant(s) does the line $y = x + 1$ pass through?
b Show that the point (1, 3) does not lie on the line $y = x + 1$.

10 Explain the process of how to check whether a point lies on a given line.

11 Using the coordinates (−1, −3), (0, −1) and (2, 3), show that a rule for the linear graph is $y = 2x - 1$.

PROBLEM SOLVING

12 Consider this pattern of squares on the grid shown.



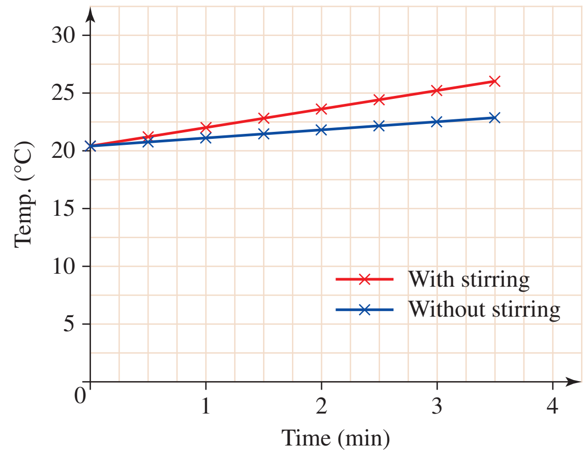
What would be the coordinates of the centre of the 20th square?

13 It is known that the mass of a certain kind of genetically modified tomato increases linearly over time. The following results were recorded.

Time, t (weeks)	1	4	6	9	16
Mass, m (grams)	6	21	31	46	81

a Plot the above points on a Cartesian plane.
b Determine the rule connecting mass with time.
c Show that the mass after 20 weeks is 101 grams.

14 As a particular chemical reaction proceeds, the temperature increases at a constant rate. The graph at right represents the same chemical reaction with and without stirring. How does stirring affect the reaction?

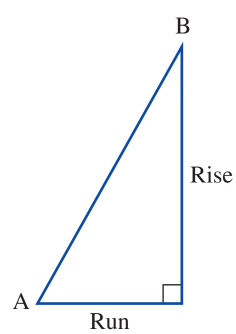


7.3 The equation of a straight line

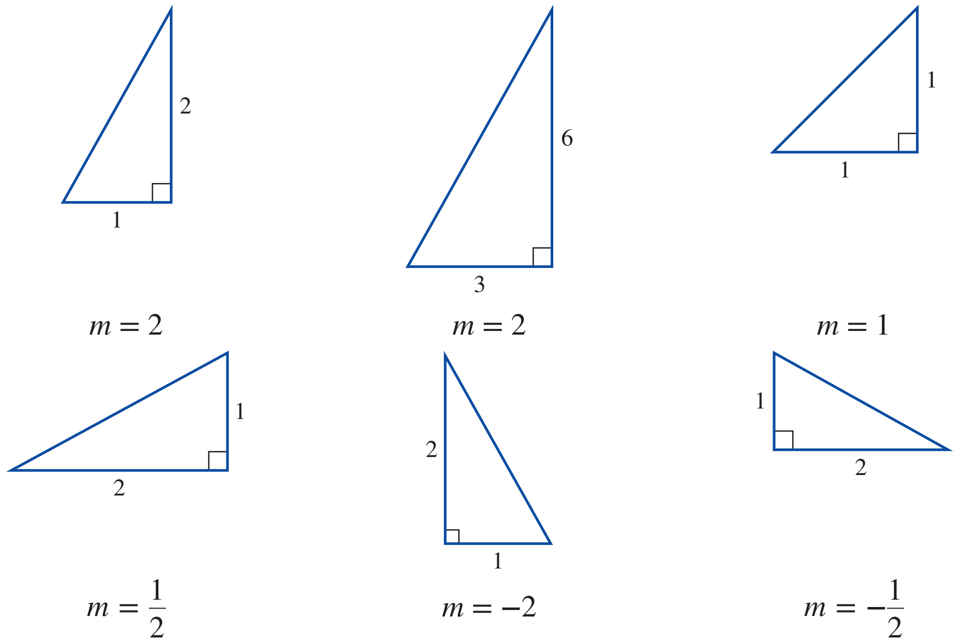
- A line goes on forever; that is, it has constant steepness or **gradient**.

The gradient (m)

- The gradient of an interval (portion of a line) is equal to the gradient of the entire line.
- The gradient of an interval AB is defined as the distance up (rise) divided by the distance across (run), and is usually given the symbol m.
- So $m = \dfrac{\text{rise}}{\text{run}}$.



- Compare these intervals and their gradients.

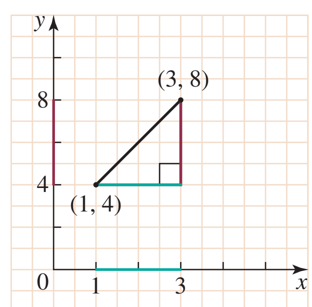


- Note that if the line is sloping downwards (from left to right), the gradient has a negative value.

Finding the gradient of a line passing through two points

- Suppose a line passes through the points (1, 4) and (3, 8), as shown in the graph at right.
- By completing a right-angled triangle, it can be seen that the rise = 8 − 4 (the difference in y-values), and the run = 3 − 1 = 2 (the difference in x-values). So

$$m = \frac{8-4}{3-1} = \frac{4}{2} = 2$$



- In general, if the line passes through the points (x_1, y_1) and (x_2, y_2), then

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

WORKED EXAMPLE 4

Find the gradient of the line passing through the points (−2, 5) and (1, 14).

THINK	WRITE
1 Let the two points be (x_1, y_1) and (x_2, y_2).	$(-2, 5) = (x_1, y_1)$, $(1, 14) = (x_2, y_2)$
2 Write the formula for gradient.	$m = \frac{y_2 - y_1}{x_2 - x_1}$

3	Substitute the coordinates of the given points into the formula and evaluate.	$= \dfrac{14 - 5}{1 - -2}$ $m = \dfrac{9}{1 + 2}$ $m = \dfrac{9}{3}$ $= 3$
4	Write the answer.	The gradient of the line passing through (−2, 5) and (1, 14) is 3.

Note: Let $(x_1, y_1) = (1, 14)$ and $(x_2, y_2) = (-2, 5)$.

The calculation becomes $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{5 - 14}{-2 - 1}$$

$$= \frac{-9}{-3}$$

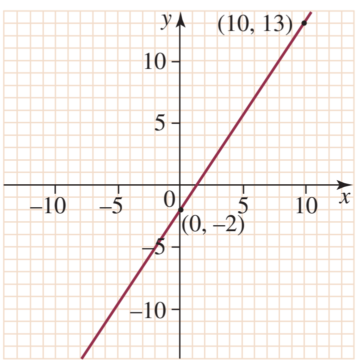
$$= 3$$

The result is the same.

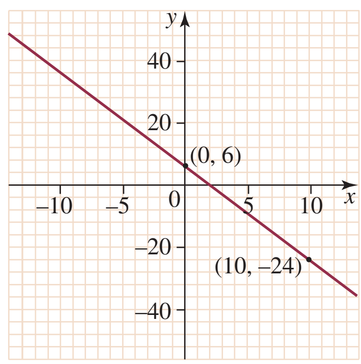
WORKED EXAMPLE 5

Calculate the gradients of the lines shown.

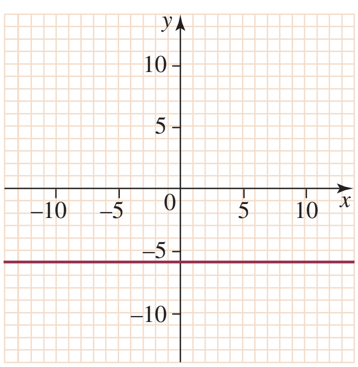
a



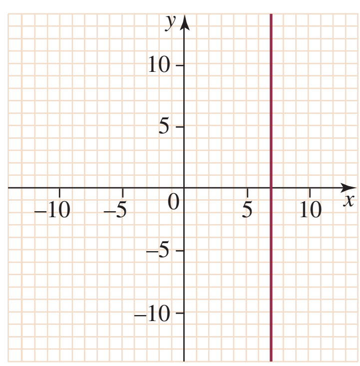
b



c



d



THINK	WRITE
a **1** Write down two points that lie on the line.	**a** Let $(x_1, y_1) = (0, -2)$ and $(x_2, y_2) = (10, 13)$. Rise $= y_2 - y_1 = 13 - -2 = 15$ Run $= x_2 - x_1 = 10 - 0 = 10$
2 Calculate the gradient by finding the ratio $\frac{\text{rise}}{\text{run}}$.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{15}{10}$ $= \frac{3}{2}$ or 1.5
b **1** Write down two points that lie on the line.	**b** Let $(x_1, y_1) = (0, 6)$ and $(x_2, y_2) = (10, -24)$. Rise $= y_2 - y_1 = -24 - 6 = -30$ Run $= x_2 - x_1 = 10 - 0 = 10$
2 Calculate the gradient.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{-30}{10}$ $= -3$
c **1** Write down two points that lie on the line.	**c** Let $(x_1, y_1) = (5, -6)$ and $(x_2, y_2) = (10, -6)$.
2 There is no rise between the two points.	Rise $= y_2 - y_1$ $= -6 - -6 = 0$ Run $= x_2 - x_1$ $= 10 - 5 = 5$
3 Calculate the gradient. Note that the gradient of a horizontal line is always zero. The line has no slope.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{0}{5}$ $= 0$
d **1** Write down two points that lie on the line.	**d** Let $(x_1, y_1) = (7, 10)$ and $(x_2, y_2) = (7, -3)$.
2 The vertical distance between the selected points is 13 units. There is no run between the two points.	Rise $= y_2 - y_1 = -3 - 10 = 13$ Run $= x_2 - x_1 = 7 - 7 = 0$
3 Calculate the gradient. *Note:* The gradient of a vertical line is always undefined.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{13}{0}$ undefined

ONLINE PAGE PROOFS

Finding the gradient of a straight line from its rule

- When an equation is written in the form $y = mx + c$, m is the value of the gradient. For example, consider the line with equation $y = 3x + 1$. The gradient is 3.
- To confirm this, find the gradient using the formula.
  Two points that lie on the line $y = 3x + 1$ are (0, 1) and (5, 16).

$$\text{Gradient} = \frac{16 - 1}{5 - 0} = \frac{15}{5} = 3$$

WORKED EXAMPLE 6

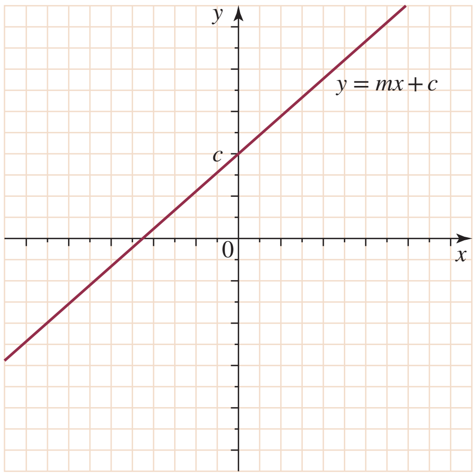
Find the gradients of the straight lines whose rules are given.

a $y = -2x + 3$ **b** $2y - 3x = 6$ **c** $y = 4$

THINK	WRITE
a The equation is the form $y = mx + c$, so the gradient is the coefficient of x.	**a** $y = -2x + 3$ $m = -2$
b 1 First rearrange the given rule so that it is in the form $y = mx + c$. (Add $3x$ to both sides, then divide both sides by 2.)	**b** $2y - 3x = 6$ $2y = 6 + 3x$ $y = \frac{6}{2} + \frac{3}{2}x$ $y = \frac{3}{2}x + 3$
2 Write the value of the gradient.	$m = \frac{3}{2}$
c 1 Rewrite the equation in the form $y = mx + c$.	**c** $y = 4$
2 Write the value of the gradient.	$m = 0$

The *y*-intercept

- For the line given by $y = mx + c$, when $x = 0$, $y = c$.
- The line passes through the point $(0, c)$. This is the point where the graph cuts the y-axis.
- The point where the graph cuts the y-axis is called the ***y*-intercept**.
- In this case the y-intercept is $(0, c)$, often simply called c.
- The y-intercept of any line is easily found by substituting 0 for x and calculating the y-value.
- $y = mx + c$ is called the 'gradient–intercept form' of the equation of a line, because it plainly displays the gradient (m) and the y-intercept (c).



WORKED EXAMPLE 7

Find the y-intercepts of the lines whose linear rules are given, and hence state the coordinates of the y-intercept.

a $y = -4x + 7$
b $5y + 2x = 10$
c $y = 2x$
d $y = -8$

THINK	WRITE
a The rule is in the gradient–intercept form, $y = mx + c$. The y-intercept is the value of c. State the coordinates.	**a** $y = -4x + 7$ $c = 7$ y-intercept: (0, 7)
b 1 To find the y-intercept, substitute $x = 0$ into the equation.	**b** $5y + 2x = 10$ $5y + 2(0) = 10$
2 Solve for y.	$5y = 10$ $y = 2$
3 Write the coordinates of the y-intercept.	y-intercept: (0, 2)
c The rule is in the gradient–intercept form, $y = mx + c$. The y-intercept is the value of c. State the coordinates.	**c** $y = 2x$ $c = 0$ y-intercept: (0, 0)
d The rule is in the form $y = mx + c$. State the coordinates.	**d** $y = -8$ $y = 0x - 8$ $c = -8$ y-intercept: (0, −8)

Exercise 7.3 The equation of a straight line

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–e, 3a–f, 4, 5a–f, 6–11, 15–16	Questions: 1d–i, 2c–f, 3e–j, 4, 5c–j, 6, 7b–e, 8, 9–12, 15–17	Questions: 1g–l, 2e–i, 3g–l, 4, 5f–l, 6, 7c–e, 8–19

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REFLECTION
Why is the y-intercept of a graph found by substituting $x = 0$ into the equation?

FLUENCY

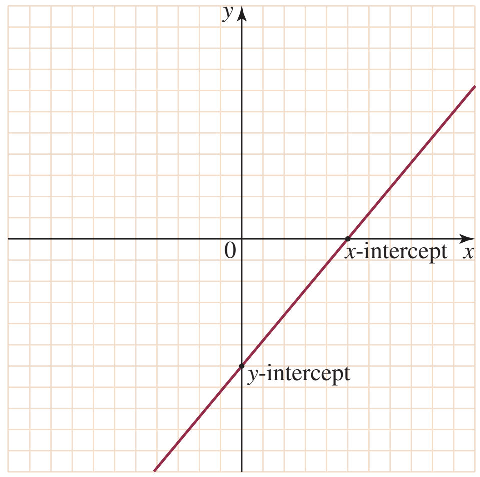
1 **WE4** Find the gradients of the lines passing through the following pairs of points.

a (2, 10) and (4, 22)
b (1, −2) and (3, −10)
c (−3, 0) and (7, 0)
d (−4, −7) and (1, −1)
e (0, 4) and (4, −4.8)
f (−2, 122) and (1, −13)
g (2, 3) and (17, 3)
h (−2, 2) and (2, 2.4)
i (1, −5) and (5, −15.4)
j (−12, −7) and (8.4, −7)
k (−2, −17.7) and (0, 0.3)
l (−3, 3.4) and (5, 2.6)

7.4 Sketching linear graphs

The x- and y-intercept method

- To use this method, the x-intercept (where the line crosses the x-axis and $y = 0$) and the y-intercept (where the line crosses the y-axis and $x = 0$) must be known.
- The line is drawn by locating each intercept, then drawing a straight line through those points.
- If both intercepts are at the origin, another point is needed to sketch the line.



WORKED EXAMPLE 8

Using the x- and y-intercept method, sketch the graphs of:

a $2y + 3x = 6$ **b** $y = \frac{4}{5}x + 5$ **c** $y = 2x$

THINK / **WRITE/DRAW**

a 1 Write the rule.

a $2y + 3x = 6$

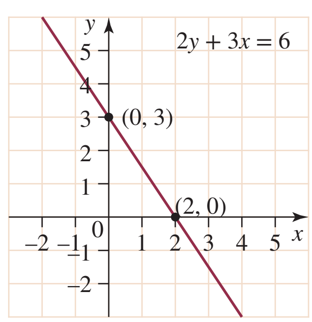
2 To find the y-intercept, let $x = 0$. Write the coordinates of the y-intercept.

$x = 0$: $2y + 3 \times 0 = 6$
$2y = 6$
$y = 3$
y-intercept: $(0, 3)$

3 To find the x-intercept, let $y = 0$. Write the coordinates of the x-intercept.

$y = 0$: $2 \times 0 + 3x = 6$
$3x = 6$
$x = 2$
x-intercept: $(2, 0)$

4 Plot and label the x- and y-intercepts on a set of axes and rule a straight line through them. Label the graph.



b 1 Write the rule.

b $y = \frac{4}{5}x + 5$

2 The rule is in the form $y = mx + c$, so the y-intercept is the value of c.

$c = 5$
y-intercept: $(0, 5)$

3 To find the x-intercept, let $y = 0$.
Write the coordinates of the x-intercept.

$$y = \frac{4}{5}x + 5$$

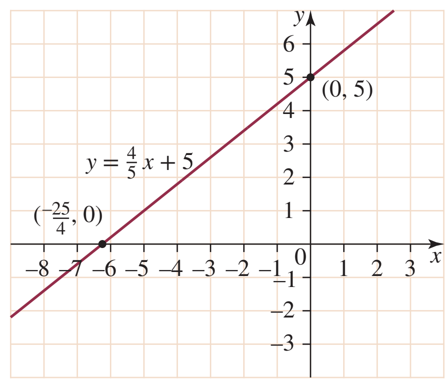
$y = 0$: $0 = \frac{4}{5}x + 5$

$$-5 = \frac{4}{5}x$$

$$x = -\frac{25}{4}$$

x-intercept: $\left(-\frac{25}{4}, 0\right)$

4 Plot and label the intercepts on a set of axes and rule a straight line through them. Label the graph.



c **1** Write the rule.

c $y = 2x$

2 To find the y-intercept, let $x = 0$.
Write the coordinates of the y-intercept.

$x = 0$: $y = 2 \times 0$
$= 0$

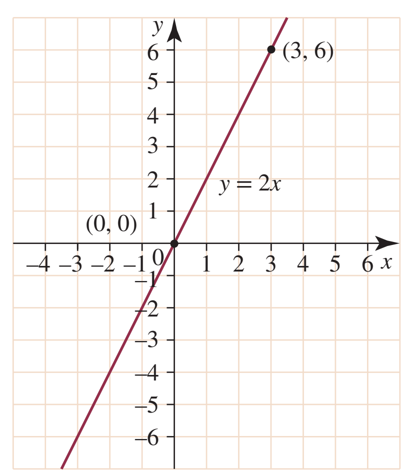
y-intercept: $(0, 0)$

3 The x- and y-intercepts are the same point, $(0, 0)$, so one more point is required. Choose any value for x, such as $x = 3$. Substitute and write the coordinates of the point.

$x = 3$: $y = 2 \times 3$
$= 6$

Another point: $(3, 6)$

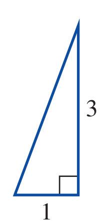
4 Plot the points, then rule and label the graph. Label the graph.



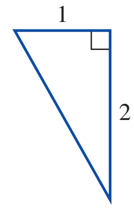
The gradient–intercept method

- To use this method, the gradient and the y-intercept must be known.
- The line is drawn by plotting the y-intercept, then drawing a line with the correct gradient through that point.

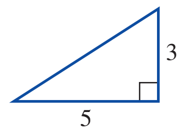
  Note:
  – A line interval of gradient 3 $\left(= \frac{3}{1}\right)$ can be drawn with a rise of 3 and a run of 1.



– Similarly, a line interval with a gradient of $-2 \left(= \frac{-2}{1}\right)$ can be shown as an interval sloping downwards.



– A line interval with a gradient of $\frac{3}{5}$ can be shown with rise = 3 and run = 5.



WORKED EXAMPLE 9

Using the gradient–intercept method, sketch the graphs of:

a $y = \frac{3}{4}x + 2$

b $4x + 2y = 3$

THINK

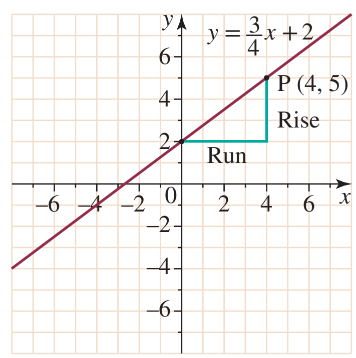
a 1 From the equation, the y-intercept is 2. Plot the point (0, 2).

2 From the equation, the gradient is $\frac{3}{4}$, so $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$. From (0, 2), run 4 units and rise 3 units. Mark the point P (4, 5).

3 Draw a line through (0, 2) and P (4, 5). Label the graph.

WRITE/DRAW

a



THINK

b 1 Write the rule in gradient–intercept form: $y = mx + c$.

From the equation, $m = -2$, $c = \frac{3}{2}$. Plot the point $(0, \frac{3}{2})$.

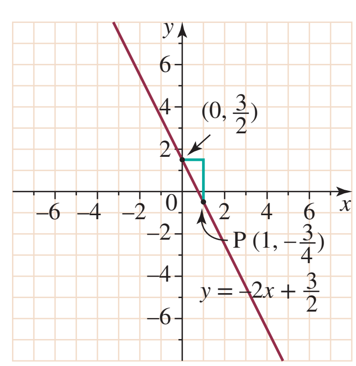
2 The gradient is -2, so $\frac{\text{rise}}{\text{run}} = \frac{-2}{1}$.

From $(0, \frac{3}{2})$, run 1 units and rise -2 units (i.e. go down 2 units). Mark the point P $(1, -\frac{1}{2})$.

3 Draw a line through $(0, \frac{3}{2})$ and P $(1, -\frac{1}{2})$. Label the graph.

WRITE/DRAW

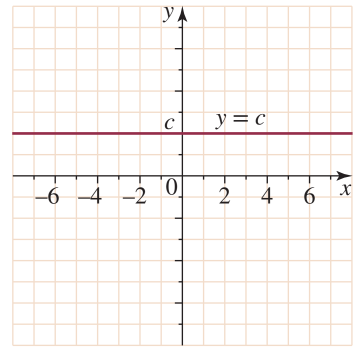
b
$$4x + 2y = 3$$
$$2y = 3 - 4x$$
$$y = \frac{3}{2} - 2x$$
$$y = -2x + \frac{3}{2}$$



Vertical and horizontal lines

$y = c$

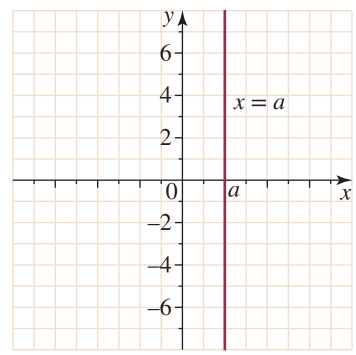
- $y = c$ is the same as $y = 0x + c$.
- This is a line with gradient 0 and y-intercept c.
- As a fraction, $0 = \frac{0}{3}, \frac{0}{4}$ and so on; therefore, a line with gradient of 0 has a rise of 0 and a run of any length except 0. This is a horizontal line.
- Using a table to find points on the line $y = c$ gives:



x	–2	0	2	4
y	c	c	c	c

$x = a$

- This equation implies that $x = a$, no matter what value y may take.
- A table of values looks like this:



x	a	a	a	a
y	–2	0	2	4

Plotting these points gives a vertical line, as shown at right.

- The run of the graph is 0, so using the formula $m = \frac{\text{rise}}{\text{run}}$ involves dividing by zero, which cannot be done. The gradient is said to be **undefined**.

WORKED EXAMPLE 10

a **On a pair of axes, sketch the graphs of:**
i $x = -3$
ii $y = 4$.
b **Label the point of intersection of the two lines.**

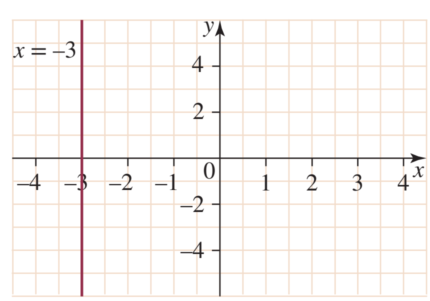
THINK

a i 1 The line $x = -3$ is in the form $x = a$. This is a vertical line.

2 Rule the vertical line where $x = -3$. Label the graph.

WRITE/DRAW

a

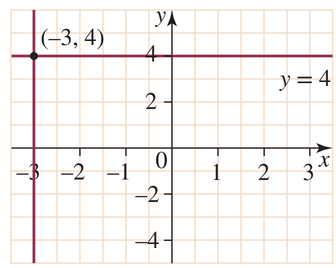


ii 1 The line $y = 4$ is in the form $y = c$. This is a horizontal line.

2 Rule the horizontal line where $y = 4$. Label the graph.

b The lines intersect at $(-3, 4)$.

b



Exercise 7.4 Sketching linear graphs

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–9, 13	Questions: 1–11, 13, 14	Questions: 1–15

Individual pathway interactivity int-4504 eBook plus

REFLECTION
Why are gradients of vertical lines undefined?

eBook plus

Digital docs
SkillSHEET
Graphing linear equations using intercepts
doc-10842
SkillSHEET
Solving linear equations that arise when finding intercepts
doc-10843

FLUENCY

1 WE8 Sketch the graphs of the following by finding the x- and y-intercepts.

a $5y - 4x = 20$ b $y = x + 2$ c $y = -3x + 6$ d $3y + 4x = -12$

e $y = 2x - 4$ f $x - y = 5$ g $x + y = 4$ h $2y + 7x - 8 = 0$

2 WE9 Sketch the graphs of the following using the gradient–intercept method.

a $y = x - 7$ b $y = 2x + 1$ c $y = 2x + 2$ d $y = -2x + 2$

e $y = \frac{1}{2}x - 1$ f $y = 4 - x$ g $y = \frac{5}{4}x + 5$ h $y = -x - 10$

3 WE10 Sketch the graphs of the following.

a $y = 4$ b $y = -3$ c $y = -12.5$ d $y = \frac{4}{5}$

4 Sketch the graphs of the following.

a $x = 2$ b $x = -6$ c $x = -2.5$ d $x = \frac{3}{4}$

5 Sketch the graphs of the following.

a $y = 3x$ b $y = -2x$ c $y = \frac{3}{4}x$ d $y = -\frac{1}{3}x$

6 MC a Which of the following statements about the rule $y = 4$ is not true?

A The gradient $m = 0$.

B The y-intercept is at $(0, 4)$.

C The graph is parallel to the x-axis.

D The point $(4, 2)$ lies on this graph.

b Which of the following statements is not true about the rule $y = -\frac{3}{5}x$?

A The graph passes through the origin.

B The gradient $m = -\frac{3}{5}$.

C The x-intercept is at $x = 0$.

D The graph can be sketched using the x- and the y-intercept method.