10.4 Graphing Quadratic Equations

10.4 OBJECTIVE

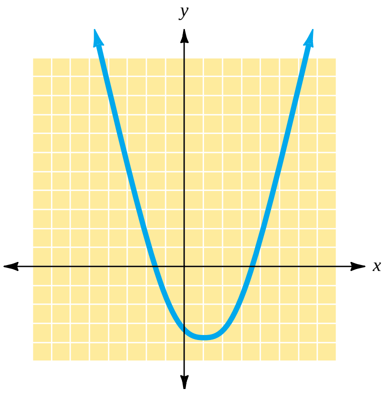
1. Graph a quadratic equation by plotting points

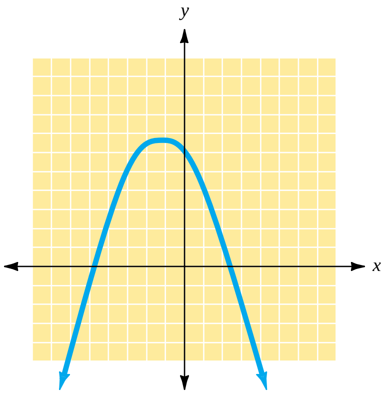
In Section 6.3 you learned to graph first-degree equations. Similar methods will allow you to graph quadratic equations of the form

$$y = ax^2 + bx + c \qquad a \neq 0$$

The first thing you will notice is that the graph of an equation in this form is not a straight line. The graph is always the curve called a **parabola.**

Here are some examples:





To graph quadratic equations, start by finding solutions for the equation. We begin by completing a table of values. This is done by choosing any convenient values for x. Then use the given equation to compute the corresponding values for y, as Example 1 illustrates.

Completing a Table of Values

If $y = x^2$, complete the ordered pairs to form solutions. Then show these results in a table of values.

$(-2, \ \ ), (-1, \ \ ), (0, \ \ ), (1, \ \ ), (2, \ \ )$

For example, to complete the pair (−2,), substitute −2 for x in the given equation.

$$y = (-2)^2 = 4$$

NOTE Remember that a solution is a pair of values that makes the equation a true statement.

So (−2, 4) is a solution.

Substituting the other values for x in the same manner, we have the following table of values for $y = x^2$:

x	y
−2	4
−1	1
0	0
1	1
2	4

CHECK YOURSELF 1

If $y = x^2 + 2$, complete the ordered pairs to form solutions and form a table of values.

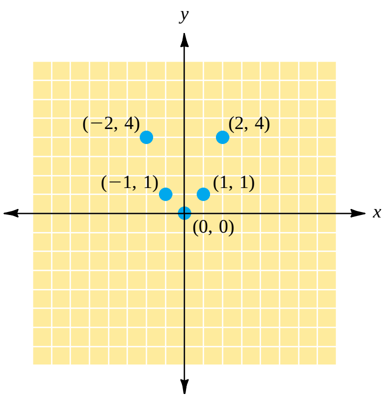
(−2,), (−1,), (0,), (1,), (2,)

We can now plot points in the cartesian coordinate system that correspond to solutions to the equation.

Plotting Some Solution Points

Plot the points from the table of values corresponding to $y = x^2$ from Example 1.

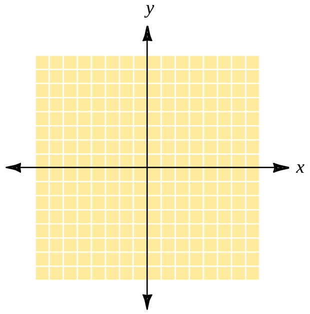
x	y
−2	4
−1	1
0	0
1	1
2	4



Notice that the y axis acts as a mirror. Do you see that any point graphed in quadrant I will be "reflected" in quadrant II?

CHECK YOURSELF 2

Plot the points from the table of values formed in Check Yourself 1.



The graph of the equation can be drawn by joining the points with a smooth curve.

Example 3

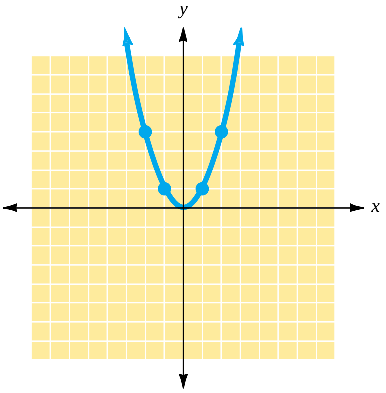
Completing the Graph of the Solution Set

Draw the graph of $y = x^2$.

We can now draw a smooth curve between the points found in Example 2 to form the graph of $y = x^2$.

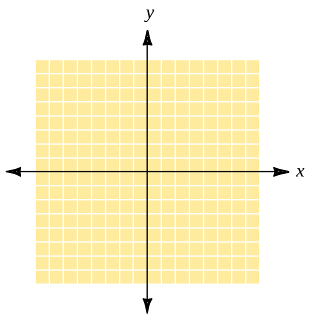
NOTE As we mentioned earlier, the graph must be the curve called a parabola.

NOTE Notice that a parabola *does* **not** come to a point.



CHECK YOURSELF 3

Draw a smooth curve between the points plotted in the Check Yourself 2 exercise.



You can use any convenient values for x in forming your table of values. You should use as many pairs as are necessary to get the correct shape of the graph (a parabola).

Example 4

Graphing the Solution Set

Graph $y = x^2 - 2x$. Use values of x between -1 and 3.

First, determine solutions for the equation. For instance, if $x = -1$,

$$
\begin{aligned}
y &= (-1)^2 - 2(-1) \\
&= 1 + 2 \\
&= 3
\end{aligned}
$$

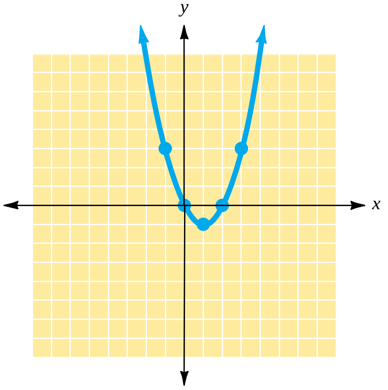
then $(-1, 3)$ is a solution for the given equation.

Substituting the other values for x, we can form the table of values shown below. We then plot the corresponding points and draw a smooth curve to form our graph.

The graph of $y = x^2 - 2x$.

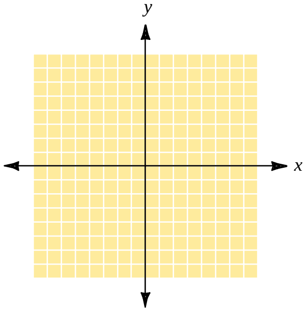
NOTE Any values can be substituted for x in the original equation.

x	y
-1	3
0	0
1	-1
2	0
3	3



CHECK YOURSELF 4

Graph $y = x^2 + 4x$. Use values of x between -4 and 0.



x	y

Choosing values for x is also a valid method of graphing a quadratic equation that contains a constant term.

Example 5

Graphing the Solution Set

Graph $y = x^2 - x - 2$. Use values of x between -2 and 3. We'll show the computation for two of the solutions.

If $x = -2$:

$$y = (-2)^2 - (-2) - 2$$
$$= 4 + 2 - 2$$
$$= 4$$

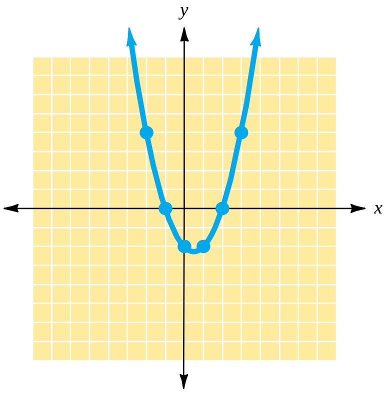
If $x = 3$:

$$y = 3^2 - 3 - 2$$
$$= 9 - 3 - 2$$
$$= 4$$

You should substitute the remaining values for x into the given equation to verify the other solutions shown in the table of values below.

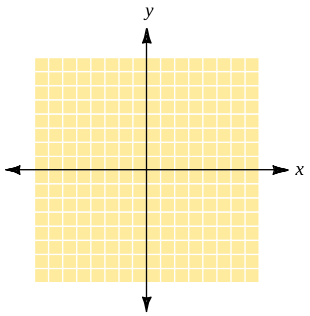
The graph of $y = x^2 - x - 2$.

x	y
-2	4
-1	0
0	-2
1	-2
2	0
3	4



CHECK YOURSELF 5

Graph $y = x^2 - 4x + 3$. Use values of x between -1 and 4.



x	y

In Example 6, the graph looks significantly different from previous graphs.

Example 6

Graphing the Solution Set

Graph $y = -x^2 + 3$. Use x values between -2 and 2.

Again we'll show two computations.

NOTE $-(-2)^2 = -4$

If $x = -2$:

$$y = -(-2)^2 + 3 = -4 + 3 = -1$$

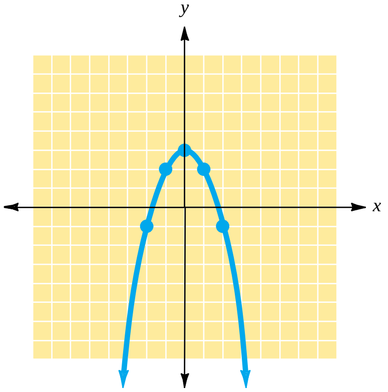
If $x = 1$:

$$y = -(1)^2 + 3 = -1 + 3 = 2$$

Verify the remainder of the solutions shown in the table of values below for yourself.

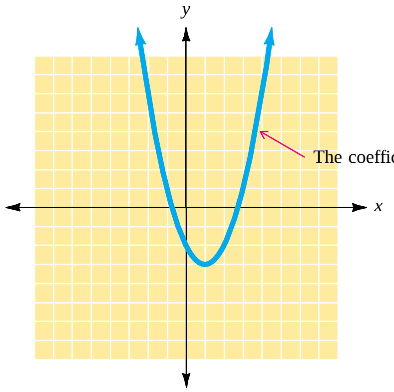
The graph of $y = -x^2 + 3$.

x	y
−2	−1
−1	2
0	3
1	2
2	−1

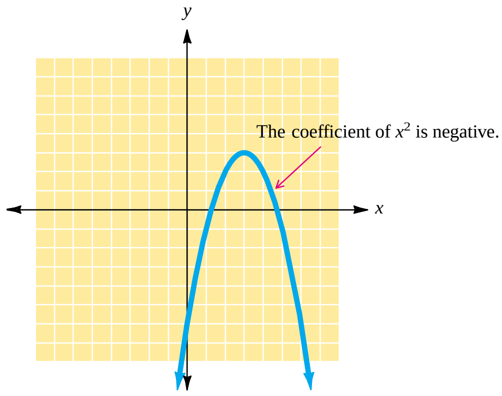


There is an important difference between this graph and the others we have seen. This time the parabola opens downward! Can you guess why? The answer is in the coefficient of the x^2 term.

If the coefficient of x^2 is *positive,* the parabola opens *upward.*

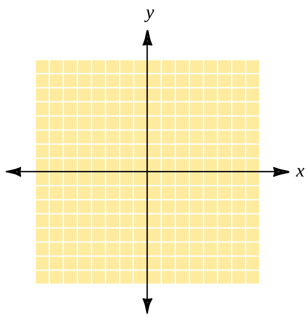


If the coefficient of x^2 is *negative,* the parabola opens *downward.*



CHECK YOURSELF 6

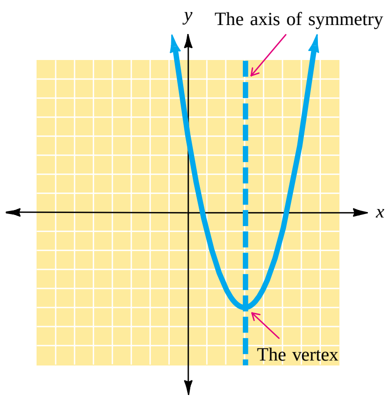
Graph $y = -x^2 - 2x$. Use x values between -3 and 1.



x	*y*

There are two other terms we would like to introduce before closing this section on graphing quadratic equations. As you may have noticed, all the parabolas that we graphed are symmetric about a vertical line. This is called the **axis of symmetry** for the parabola.

The point at which the parabola intersects that vertical line (this will be the lowest—or the highest—point on the parabola) is called the **vertex.** You'll learn more about finding the axis of symmetry and the vertex of a parabola in your next course in algebra.

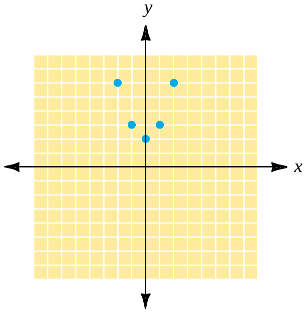


CHECK YOURSELF ANSWERS

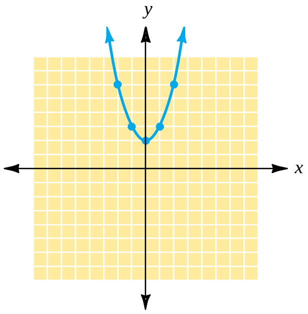
1.

x	y
-2	6
-1	3
0	2
1	3
2	6

2.

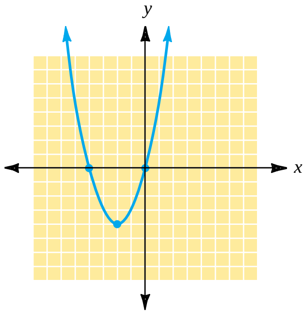


3. $y = x^2 + 2$



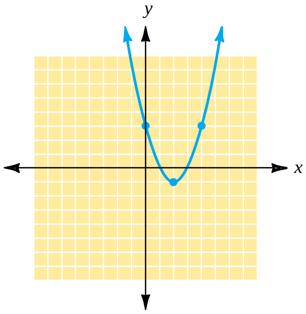
4. $y = x^2 + 4x$

x	y
-4	0
-3	-3
-2	-4
-1	-3
0	0



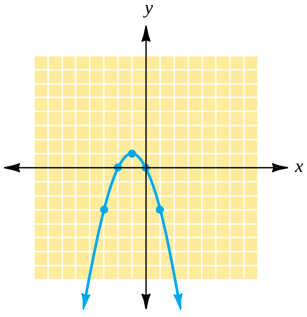
5. $y = x^2 - 4x + 3$

x	y
-1	8
0	3
1	0
2	-1
3	0
4	3



6. $y = -x^2 - 2x$

x	y
-3	-3
-2	0
-1	1
0	0
1	-3



Name ____________________

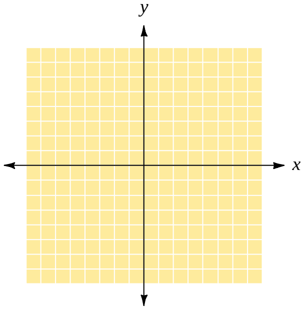
Section __________ Date __________

10.4 Exercises

Graph each of the following quadratic equations after completing the given table of values.

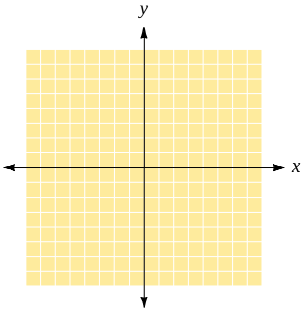
1. $y = x^2 + 1$

x	y
−2	
−1	
0	
1	
2	



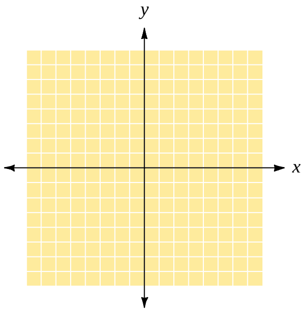
2. $y = x^2 - 2$

x	y
−2	
−1	
0	
1	
2	



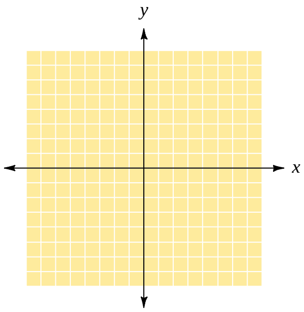
3. $y = x^2 - 4$

x	y
−2	
−1	
0	
1	
2	



4. $y = x^2 + 3$

x	y
−2	
−1	
0	
1	
2	



ANSWERS

1. ____________________
2. ____________________
3. ____________________
4. ____________________

ANSWERS

5. ______

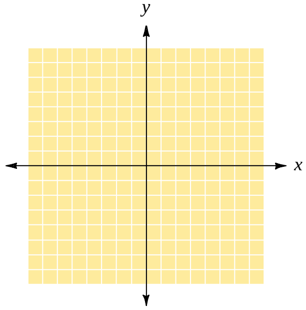
6. ______

7. ______

8. ______

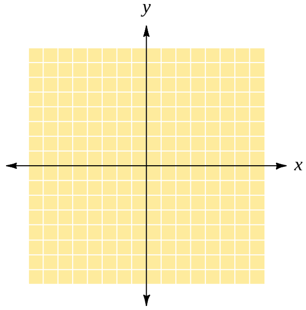
5. $y = x^2 - 4x$

x	y
0	
1	
2	
3	
4	



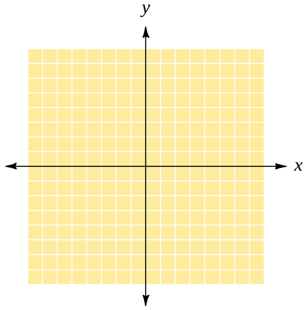
6. $y = x^2 + 2x$

x	y
−3	
−2	
−1	
0	
1	



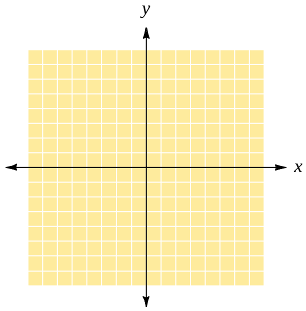
7. $y = x^2 + x$

x	y
−2	
−1	
0	
1	
2	



8. $y = x^2 - 3x$

x	y
−1	
0	
1	
2	
3	



ANSWERS

9. ______

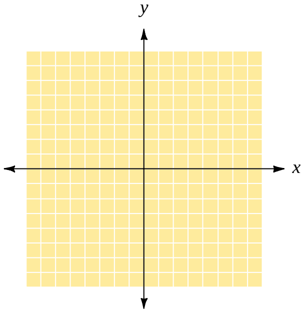
10. ______

11. ______

12. ______

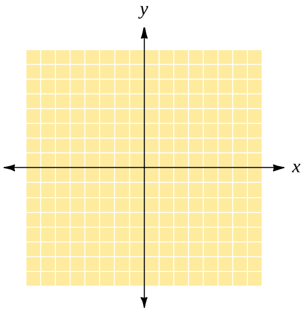
9. $y = x^2 - 2x - 3$

x	y
−1	
0	
1	
2	
3	



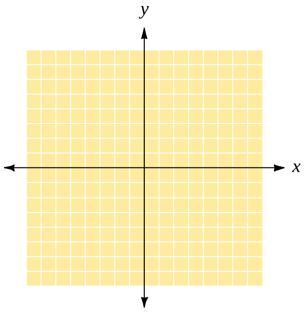
10. $y = x^2 - 5x + 6$

x	y
0	
1	
2	
3	
4	



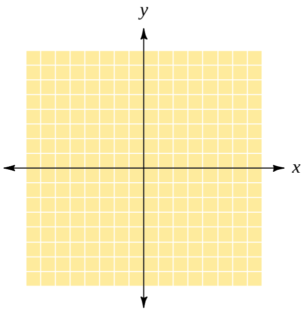
11. $y = x^2 - x - 6$

x	y
−1	
0	
1	
2	
3	



12. $y = x^2 + 3x - 4$

x	y
−4	
−3	
−2	
−1	
0	



ANSWERS

13. ______

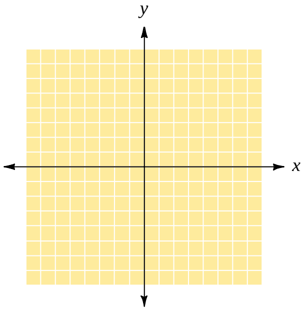
14. ______

15. ______

16. ______

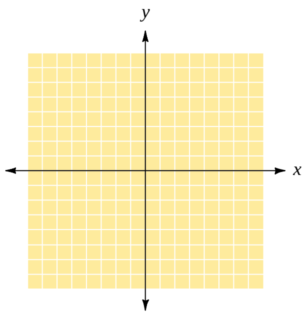
13. $y = -x^2 + 2$

x	y
-2	
-1	
0	
1	
2	



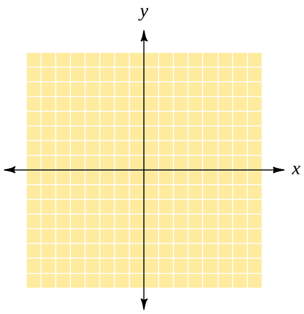
14. $y = -x^2 - 2$

x	y
-2	
-1	
0	
1	
2	



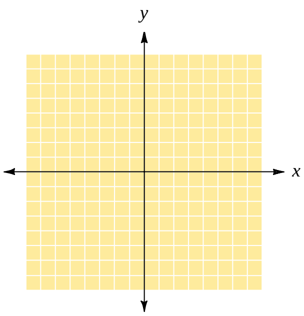
15. $y = -x^2 - 4x$

x	y
-4	
-3	
-2	
-1	
0	



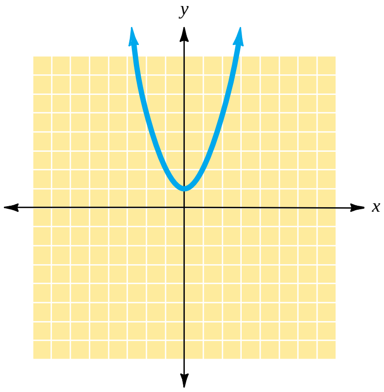
16. $y = -x^2 + 2x$

x	y
-1	
0	
1	
2	
3	



Match each graph with the correct equation on the right.

17.



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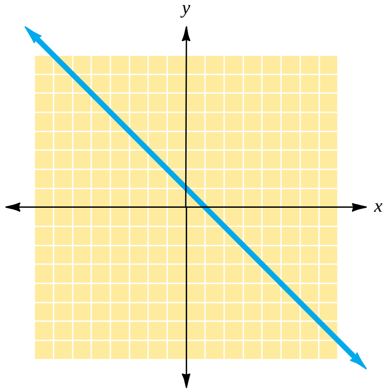

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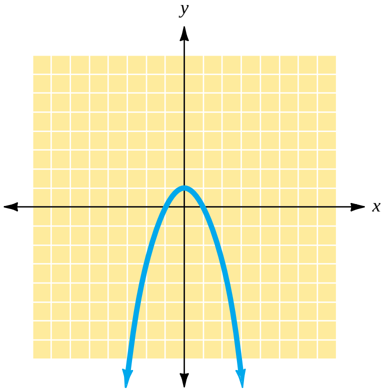

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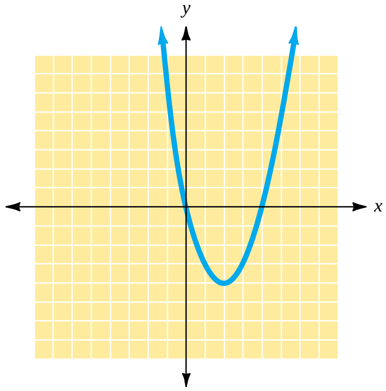

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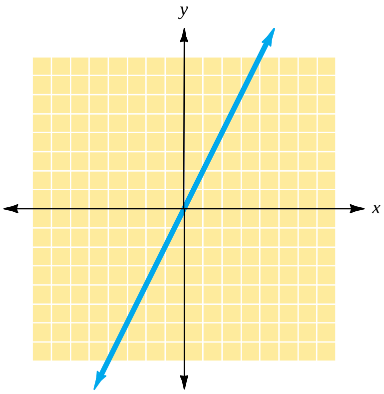

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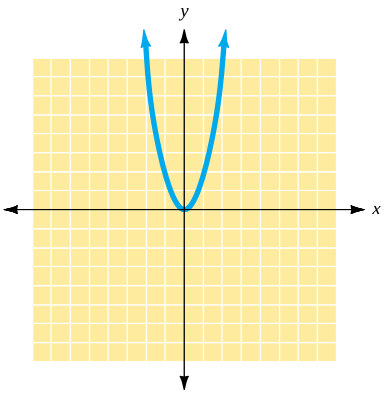

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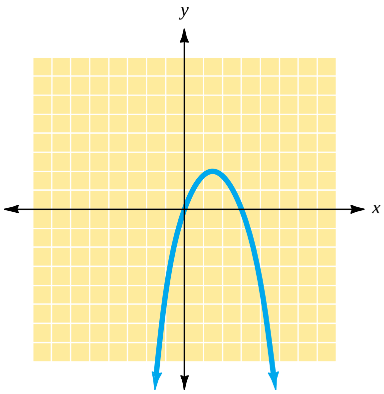

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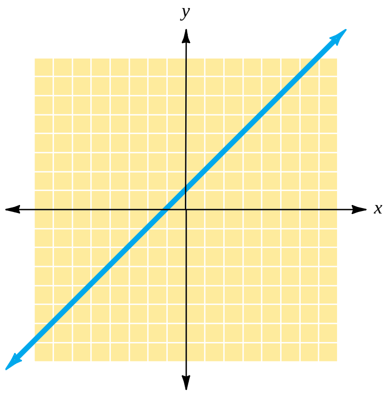


17. ______

18. ______

19. ______

20. ______

21. ______

22. ______

23. ______

24. ______

(a) $y = -x^2 + 1$

(b) $y = 2x$

(c) $y = x^2 - 4x$

(d) $y = -x + 1$

(e) $y = -x^2 + 3x$

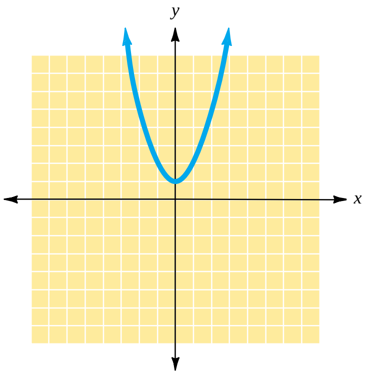
(f) $y = x^2 + 1$

(g) $y = x + 1$

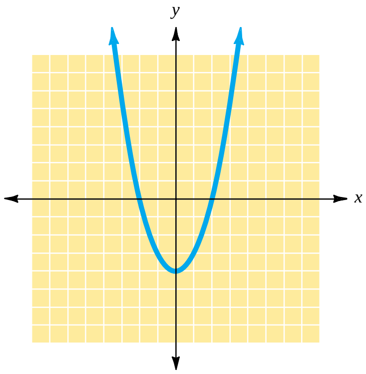
(h) $y = 2x^2$

Answers

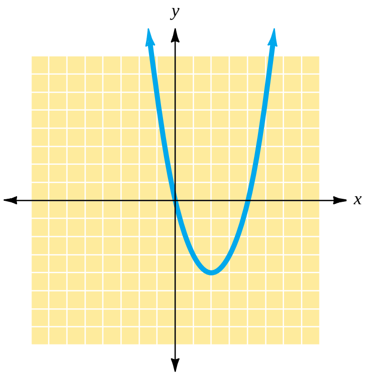
1. $y = x^2 + 1$



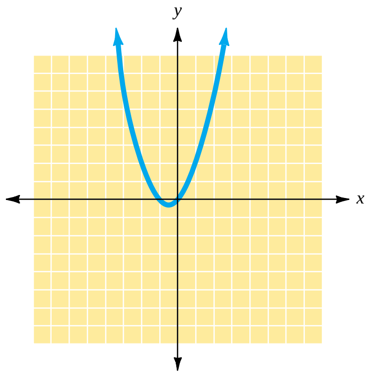
3. $y = x^2 - 4$



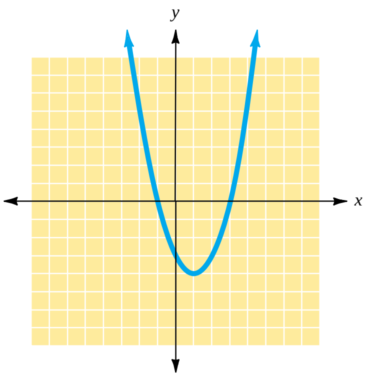
5. $y = x^2 - 4x$



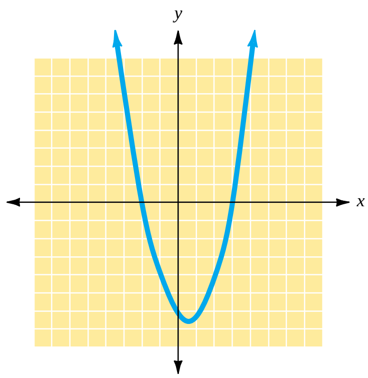
7. $y = x^2 + x$



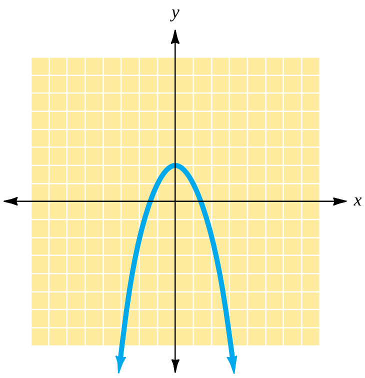
9. $y = x^2 - 2x - 3$



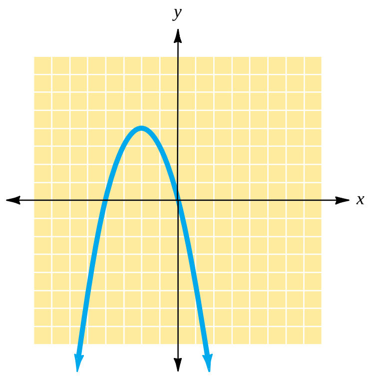
11. $y = x^2 - x - 6$



13. $y = -x^2 + 2$



15. $y = -x^2 - 4x$



17. f **19.** a **21.** b **23.** e