Maths Learning Service: Revision ***Mathematics IA***

Function Notation

Mathematics IMA

A **function** is a rule for calculating a *single* value $y = f(x)$ from an input value x. (**Note:** "$f(x)$" does *not* mean "$f \times x$".)

Examples:

(1) Consider the rule $y = f(x) = 2x + 3$.

If $x = 1$ then $f(1) = 2 \times 1 + 3 = 5$
If $x = -3$ then $f(-3) = 2 \times (-3) + 3 = -3$ etc.

Recall that the points (x, y) or $(x, f(x))$ satisfying this rule lie on a straight line. We say that the graph of the function $f(x) = 2x + 3$ is a straight line.

(2) Consider the rule $f(x) = x^2 + 2x$.

Again recalling earlier work, we know that the graph of this function is a parabola.

If $x = 1$ then $f(1) = 1^2 + 2 \times 1 = 3$
If $x = -3$ then $f(-3) = (-3)^2 + 2 \times (-3) = 3$ etc.

(3) Function notation allows us to input algebraic symbols and formulae as well as numbers. If $f(x) = x^2 - 1$, then

(a) $f(a) = a^2 - 1$

(b) $f(x + h) = (x + h)^2 - 1$

(c) $f(x^2) = (x^2)^2 - 1 = x^4 - 1$

(d) $f(\sqrt{x+1}) = (\sqrt{x+1})^2 - 1 = x$

(4) Consider two functions $f(x) = x^2$ and $g(x) = x + 1$. The following **composite functions** can be formed

(a) $f(g(x)) = f(x + 1) = (x + 1)^2$
(b) $g(f(x)) = g(x^2) = x^2 + 1$
(c) $f(f(x)) = f(x^2) = (x^2)^2 = x^4$
(d) $g(g(x)) = g(x + 1) = (x + 1) + 1 = x + 2$

Note: The concept of composite functions is useful for understanding the Chain Rule of differentiation and inverse functions.

Exercises

(1) If $f(x) = 3x + 1$, then find the following

(a) $f(1)$ (b) $f(-2)$ (c) $f(x + \psi)$ (d) $f(x^3)$

(2) If $f(x) = 3x^2 + x - 2$, then find the following

(a) $f(-1)$ (b) $f(x^2)$ (c) $\dfrac{f(x+h) - f(x)}{h}$

(3) Find $f(g(x))$, $g(f(x))$, $f(f(x))$ and $g(g(x))$ for the following.

(a) $f(x) = \dfrac{1}{x}, \quad g(x) = x + 3$

(b) $f(x) = \dfrac{x}{2}, \quad g(x) = 2x$

(c) $f(x) = x^2, \quad g(x) = \sqrt{x} \quad (x \geq 0)$

Answers to Exercises

(1) (a) 4 (b) -5 (c) $3(x + \psi) + 1$ (d) $3x^3 + 1$

(2) (a) 0 (b) $3x^4 + x^2 - 2$ (c) $\dfrac{6xh + h^2 + h}{h} = 6x + 3h + 1$

(3) (a) $\dfrac{1}{x+3}, \ \dfrac{1}{x} + 3, \ x, \ x + 6$ (b) $x, \ x, \ \dfrac{x}{4}, \ 4x$ (c) $x, \ x, \ x^4, \ \sqrt[4]{x}$