INTEGRATING EXPONENTIAL EXPRESSIONS

$$\int \left(e^{kx+d}\right)dx = \frac{1}{k}e^{kx+d} + c$$

To antidifferentiate exponential functions whose powers are linear expressions:

Step 1: Re-write the given expression.

Step 2: Divide the expression by the coefficient of x (number and sign in front of x).

Step 3: Add the constant of integration.

Do NOT raise the power on the exponential term by 1.

EXAMPLE 1

Integrate the following expressions:

(a) $\int 5e^{2x-1}\,dx$

$$\int 5e^{2x-1}\,dx = \frac{5e^{2x-1}}{2} + c$$

(b) $\int \frac{3}{e^{5x}}\,dx$

Bring x terms to the numerator by changing the sign on the power of x:

$$\int \frac{3}{e^{5x}}\,dx = \int 3e^{-5x}\,dx$$

Antidifferentiate:

$$= -\frac{3}{5}e^{-5x} + c$$

Write the answer with positive powers:

$$= -\frac{3}{5e^{5x}} + c$$

(c) $\int \frac{e^{3x}+e^{-x}}{2e^{x}}\,dx$

Write each term in the numerator over the denominator and simplify:

$$\int \frac{e^{3x}+e^{-x}}{2e^{x}}\,dx=\int\left(\frac{e^{3x}}{2e^{x}}+\frac{e^{-x}}{2e^{x}}\right)dx=\int\left(\frac{e^{2x}}{2}+\frac{e^{-x-x}}{2}\right)dx=\int\left(\frac{e^{2x}}{2}+\frac{e^{-2x}}{2}\right)dx$$

Antidifferentiate:

$$=\frac{e^{2x}}{2\times 2}+\frac{e^{-2x}}{2\times -2}+c$$

$$=\frac{e^{2x}}{4}-\frac{e^{-2x}}{4}+c$$

Write the answer with positive powers:

$$=\frac{e^{2x}}{4}-\frac{1}{4e^{2x}}+c$$

(d) $\int \frac{xe^{1-2x}-x\sin x}{x}\,dx$

Factorise and simply by cancellation:

$$\int \frac{xe^{1-2x}-x\sin x}{x}\,dx=\int e^{1-2x}-\sin x\,dx$$

Antidifferentiate:

$$=\frac{e^{1-2x}}{-2}--\cos x+c$$

$$=\cos x-\frac{e^{1-2x}}{2}+c$$