INTEGRATING ALGEBRAIC EXPRESSIONS

To antidifferentiate algebraic expressions, we raise the power on the variable (usually x) by one, and divide the new term by the new power.

$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c, \quad n \neq -1$$

This rule applies for all algebraic expressions, including rational functions, providing that $n \neq -1$.

For Example: $\int x^3 \, dx = \frac{x^{3+1}}{3+1} + c = \frac{x^4}{4} + c$

The antiderivative of an expression consisting of the sum/difference of a series of terms is equivalent to the sum/difference of the antiderivatives of each individual term.

For Example: $\int \left(x^3 - 2x^2 + 1\right) dx = \int \left(x^3\right) dx - \int \left(2x^2\right) dx + \int 1 \, dx$

Note:

- **c** is an arbitrary constant that must be included every time an expression is antidifferentiated. The only time the arbitrary constant can be omitted is when **"an antiderivative"** is required.

- Expressions may be simplified by removing constants and placing them in front of the symbol of integration.

For Example: $\int (ax) \, dx = a \int x \, dx$

$\int (ax^3 + bx^2 + cx) \, dx = a \int x^3 \, dx + b \int x^2 \, dx + c \int x \, dx$

For example: $\int \left(2x^3\right) dx = 2 \int \left(x^3\right) dx$

INTEGRATING EXPRESSIONS – GENERAL APPROACH

Step 1: Rewrite all terms as powers on x.

For example: $\int \left(x^3 + 6\sqrt{x}\right) dx = \int \left(x^3 + 6x^{\frac{1}{2}}\right) dx$

Step 2: Bring terms involving x in the denominator (bottom of a fraction) to the top, by changing the sign on the power.

For example: $\frac{1}{x^2} = x^{-2}$. **Note:** $\frac{1}{6x^2} = \frac{x^{-2}}{6}$.

Step 3: Simplify expressions so that terms are separated by addition and subtraction and then antidifferentiate each term individually. Alternatively, reduce expressions down to 1 term only (use log and index laws).

Step 4: Antidifferentiate.

Step 5: Re-write the answer using positive powers. Bring terms with negative powers in the numerator (top of a fraction) to the bottom by changing the sign on the power.

Step 6: State restrictions on the values of x.

Note:

- VCAA will deduct 1 mark if you forget to include the "dx".
- If asked to find "an" antiderivative, no "c" is required.

Example 1
Integrate the following expressions.

(a) $\int (5t^8 - 2t^4 + t + 3)\,dt$

Raise the power on the variable (usually x) in each term by one and divide the new term by the new power.

$$\int (5t^8 - 2t^4 + t + 3)\,dt = \frac{5}{9}t^9 - \frac{2}{5}t^5 + \frac{1}{2}t^2 + 3t + c$$

(b) $\int 6x^2(x^4 - 1)\,dx$

Expand brackets:

$$\int 6x^2(x^4 - 1)\,dx = \int (6x^6 - 6x^2)\,dx$$

Raise the power on the variable (usually x) in each term by one and divide the new term by the new power.

$$= \frac{6x^7}{7} + \frac{6x^3}{3} + c$$

$$= \frac{6x^7}{7} + 2x^3 + c$$

(c) $\int \frac{x^2 + 6x + 9}{x + 3}\,dx$

Factorise and simply by cancellation:

$$\int \frac{x^2 + 6x + 9}{x + 3}\,dx = \int \frac{(x+3)^2}{(x+3)}\,dx = \int (x+3)\,dx$$

Antidifferentiate:

$$= \frac{x^2}{2} + 3x + c$$

(d) $\int \left(x^{1.4} - 5x^{1.5}\right) dx$

Raise the power on the variable (usually x) in each term by one and divide the new term by the new power.

$$\int \left(x^{1.4} - 5x^{1.5}\right) dx = \frac{x^{2.4}}{2.4} - \frac{5y^{2.5}}{2.5} + c$$

$$= \frac{5x^{2.4}}{12} - 2y^{2.5} + c$$

(e) $\int \left(\frac{4}{3x^2} + \sqrt{x}\right) dx$

Remove the square root symbol:

$$\int \left(\frac{4}{3x^2} + \sqrt{x}\right) dx = \int \left(\frac{4}{3x^2} + x^{1/2}\right) dx$$

Bring x terms to the numerator by changing the sign on the power of x:

$$= \int \left(\frac{4x^{-2}}{3} + x^{1/2}\right) dx$$

Antidifferentiate:

$$= \frac{4x^{-1}}{3 \times -1} + \frac{x^{3/2}}{3/2} + c$$

$$= \frac{-4x^{-1}}{3} + \frac{2x^{3/2}}{3} + c$$

Write the answer with positive powers:

$$= \frac{-4}{3x} + \frac{2x^{3/2}}{3} + c$$

(f) $\int\left(\frac{1}{x\sqrt{x}}\right)dx$

Remove the square root symbol:

$$\int\left(\frac{1}{x\sqrt{x}}\right)dx = \int \frac{1}{x.x^{1/2}}\,dx = \int \frac{1}{x^{3/2}}\,dx$$

Bring x terms to the numerator by changing the sign on the power of x:

$$= \int x^{-3/2}\,dx$$

Antidifferentiate:

$$= \frac{x^{-3/2+1}}{-3/2+1} + c$$

$$= \frac{x^{-1/2}}{-1/2} + x$$

Write the answer with positive powers:

$$= \frac{-2}{\sqrt{x}} + c$$

(g) $\int\left(\frac{x^4+2}{x^2}\right)dx$

Write each term in the numerator over the denominator and simplify:

$$\int\left(\frac{x^4+2}{x^2}\right)dx = \int\left(\frac{x^4}{x^2}+\frac{2}{x^2}\right)dx = \int\left(x^2+\frac{2}{x^2}\right)dx$$

Bring x terms to the numerator by changing the sign on the power of x:

$$= \int\left(x^2 + x^{-2}\right)dx$$

Antidifferentiate:

$$= \frac{x^3}{3} + \frac{x^{-1}}{-1} + c = \frac{x^3}{3} - x^{-1} + c$$

Write the answer with positive powers:

$$= \frac{x^3}{3} - \frac{1}{x} + c$$

(h) $\int x^{m-1}x^m\,dx$

Rewrite the product as a single term. As the bases are the same, we can add powers.

$$\int x^{m-1}x^m\,dx = \int x^{m-1+m}\,dx = \int x^{2m-1}\,dx$$

Raise the power on the variable (usually x) in each term by one and divide the new term by the new power.

$$= \frac{x^{2m-1+1}}{2m-1+1} + c = \frac{x^{2m}}{2m} + c$$

(i) $\int \left(\frac{2x^{1-3m}}{5x^{2m^2}}\right)dx$

Rewrite the quotient as a single term. As the bases are the same, we can subtract powers.

$$\int \left(\frac{2x^{1-3m}}{5x^{2m^2}}\right)dx = \int \left(\frac{2x^{1-3m-2m^2}}{5}\right)dx$$

Raise the power on the variable (usually x) in each term by one and divide the new term by the new power.

$$= \frac{2x^{1-3m-2m^2+1}}{5(1-3m-2m^2+1)} + c = \frac{2x^{2-3m-2m^2}}{5(2-3m-2m^2)} + c$$