INTEGRATING (AX+B)^N

Given a **linear expression** in terms of x that is raised to the power of a large number, a fraction or a negative number (excluding -1), the antiderivative is obtained by applying the following rule:

$$\int (ax+b)^n \ dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c$$

$$\int (linear\ expression)^n \ dx \ = \frac{given\ expression\ (raise\ power\ by\ 1)}{coefficient\ of\ x \ \times \ new\ power} + c$$

EXAMPLE 1

Find $\int (4x-9)^{15}\, dx$.

Solution

Raise the power on the brackets by one. Then divide the expression by the new power and the coefficient of x.

$$\int (4x-9)^{15}\, dx \ = \frac{(4x-9)^{16}}{16\times 4} + c$$

$$= \frac{(4x-9)^{16}}{64} + c$$

EXAMPLE 2

Find $\int \sqrt{2x+1}\, dx$.

Solution

Remove the square root symbol:

$$\int \sqrt{2x+1}\, dx = \int (2x+1)^{1/2}\, dx$$

Raise the power on the brackets by one. Then divide the expression by the new power and the coefficient of x.

$$= \frac{(2x+1)^{3/2}}{3/2 \times 2} + c = \frac{(2x+1)^{3/2}}{3} + c$$

EXAMPLE 3

Find $\int \frac{dx}{(3-5x)^4}$.

Solution

Bring x terms to the numerator by changing the sign on the power of x:

$$\int \frac{dx}{(3-5x)^4} = \int (3-5x)^{-4}\, dx$$

Raise the power on the brackets by one. Then divide the expression by the new power and the coefficient of x.

$$= \frac{(3-5x)^{-3}}{-3 \times -5} + c$$

$$= \frac{(3-5x)^{-3}}{15} + c$$