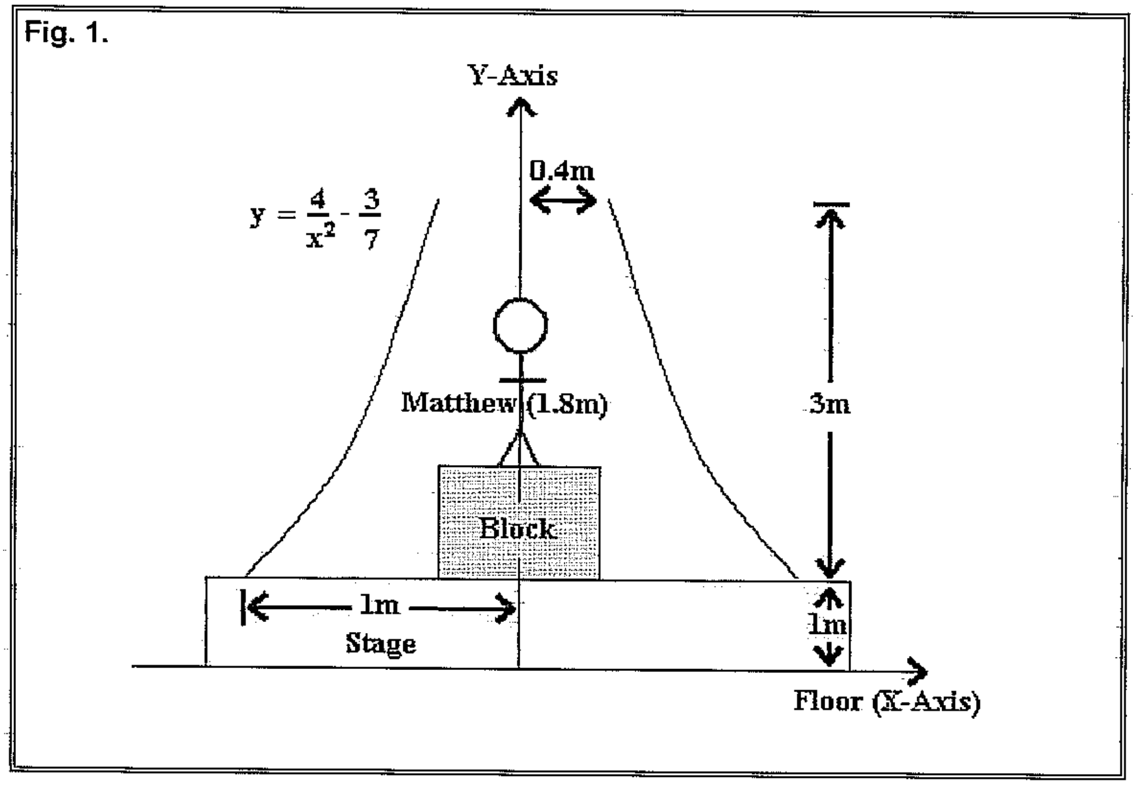
The Great Escape

Fig. 1.



(Question 1): Determining curve equation.

Floor is X-axis; vertical center Y-axis. (Dimensions see **Fig. 1**)

a. $y = \frac{a}{x^2} + b$ models shape of flask longitudinal cross-section and is found by substituting X and Y values (from dimensions) into general truncus formula; simultaneously equating to find constants a and b.

When x = 0.4m; y = 4m

$$\therefore 4 = \frac{a}{(0.4)^2} + b$$

$$\therefore 4 = \frac{a}{\left(\frac{4}{10}\right)^2} + b$$

$$\therefore 4 = \frac{a}{\left(\frac{16}{100}\right)} + b$$

$$\therefore 4 = \frac{100a}{16} + b$$

Equation (1)

When x = 1m; y = 1m

$$\therefore 1 = \frac{a}{(1)^2} + b$$

$$\therefore 1 = a + b$$

Equation (2)

(1) - (2):

$$\left(4=\frac{100a}{16}+b\right)-(1=a+b)$$

$$\Rightarrow 3=\frac{21}{4}a$$

$$\Rightarrow a=3\times\frac{4}{21}$$

$$\Rightarrow a=\frac{4}{7}$$

Substituting a = $\frac{4}{7}$:

$$1=\frac{4}{7}+b$$

$$b=\frac{7}{7}-\frac{4}{7}$$

$$b=\frac{3}{7}$$

$$\therefore y=\frac{4}{7x^2}+\frac{3}{7} \quad \{x: 0.4\text{m.} \le x \le 1\text{m.}\}$$

b. Volume of flask (L.) as function of height above floor:

Anti-differentiation and rotation around Y-axis giving volume of flask in litres **(*L*)** as function of ***y(m)*** above floor. Accurately determines flask's volume only because flasks transverse cross-section is circular.

Volume of rotation around Y-Axis:

$$V=\pi\int_b^a x^2.dy$$

- Upper limit *a = 4m;*
- Lower limit *b = 1m* ***{Stage height:*** h_s ***(m)}***

Transposition making x^2 subject:

$$y=\frac{4}{7x^2}+\frac{3}{7}$$

$$\frac{4}{7x^2}=y-\frac{3}{7}$$

$$\frac{4}{x^2}=7y-3$$

$$x^2=\frac{4}{7y-3}$$

Let V = volume (L):

$$V = \pi\int_1^4 \frac{4}{7y-3}.dy\left(m^3\right)$$

$$V = \pi\left[\frac{4}{7}\log_e(7y-3)\right]_1^4 m^3$$

$$V = \pi\left[\left(\frac{4}{7}\log_e 7(4)-3\right)-\left(\frac{4}{7}\log_e 7(1)-3\right)\right]m^3$$

$$V = \pi\left[\left(\frac{4}{7}\log_e 25\right)-\left(\frac{4}{7}\log_e 4\right)\right]m^3$$

$$V = \pi\left(\frac{4}{7}\log_e \frac{25}{4}\right)m^3$$

$$V = \frac{4\pi}{7}(\log_e 6.25)m^3$$

$$V = 3.28984m^3 \times 1000$$

$$V = 3289.8L$$

$$V = 3290L$$

General equation V_{Litres} as function of height above floor, ***Let y = h_f (m)***:

$$V(y) = \pi\left[\frac{4}{7}\log_e(7y-3)\right]_1^y m^3 \times 1000$$

$$V(y) = \frac{4000\pi}{7}\left[\log_e(7y-3)\right]_1^y L.$$

$$V(y) = \frac{4000\pi}{7}\left[\log_e(7y-3)-\log_e 4\right]L.$$

$$V(y) = \frac{4000\pi}{7}\left(\log_e\frac{(7y-3)}{4}\right)L.;\{y:1\le y\le 4\}$$

$$V(h_f) = \frac{4000\pi}{7}\left(\log_e\frac{(7h-3)}{4}\right)L.;\{h_f:1\le h_f\le 4\}$$

(Question 3):

- Matthew's height = *1.8m. (Denoted by **h_m**)*
- Escape time = *10 minutes (min.)*
- Rate water pumped into flask = *300L./min.* (Assumed water enters from bottom)
- From #2: Maximum volume = *3290L.*; Minimum volume = *0L.*
  ∴{V: 0L. ≤ V ≤ 3290L.}
- Find: Block height (**h_b**) when water level just reaches top of Matthew's head.

{h_b: 0m. ≤ h_b ≤ 1.2m.} As h_b = 1.2m. and h_m =1.8m. is height of flask top 3m.
(For these calculations, volume of Matthew and block considered negligible).

Transposing V(h) equation giving General formula for height above floor ***h_f(m)***. as a function of volume (L):

$$V(h_f) = \frac{4000\pi}{7}\left(\log_e \frac{(7h_f - 3)}{4}\right)L.;\{h_f : 1 \le h_f \le 4\}$$

$$\Rightarrow \frac{7V}{4000\pi} = \log_e \frac{(7h_f - 3)}{4}$$

$$\Rightarrow e^{\frac{7V}{4000\pi}} = \frac{(7h_f - 3)}{4}$$

$$\Rightarrow 4e^{\frac{7V}{4000\pi}} = 7h_f - 3$$

$$\Rightarrow 4e^{\frac{7V}{4000\pi}} + 3 = 7h_f$$

$$\Rightarrow h_f = \frac{1}{7}\left(4e^{\frac{7V}{4000\pi}} + 3\right)m.;\{V : 0L. \le V \le 3290L.\}$$

∴ 1m **(h_s)** is subtracted giving: Height of water in flask as function of volume.
Let h_w = height of water in flask (m):

$$\Rightarrow h_w(V) = \frac{1}{7}\left(4e^{\frac{7V}{4000\pi}} + 3\right) - 1$$

$$\Rightarrow h_w(V) = \frac{4}{7}e^{\frac{7V}{4000\pi}} + \frac{3}{7} - \frac{7}{7}$$

$$\Rightarrow h_w(V) = \frac{4}{7}e^{\frac{7V}{4000\pi}} - \frac{4}{7}$$

$$\Rightarrow h_w(V) = \frac{4}{7}\left(e^{\frac{7V}{4000\pi}} - 1\right);\{V : 0 \le V \le 3290\}$$

For given time **(t)** minutes, $V = 300t$

∴ after 10 minutes, V = 3000L

$$\therefore h_w(3000) = \frac{4}{7}\left(e^{\frac{7(3000)}{4000\pi}} - 1\right);\{V : 0 \le V \le 3290\}$$

$$\therefore h_w = 2.4675m$$

∴ ***h_b* (m):** ∴ ***h_b* =** *h_w* - h_m

∴ h_b = 2.4675 - 1.8

= 0.6675

= 0.67m

∴ Height of block required to just reach top of Matthew's head after 10min. at 300L./min. is 67cm.

(Question 4): Submergence time (T)min. – Time taken for water to overflow after reaching Matthew's head:

a. Rate of water pumped into flask to overflow after 10min:

- V = 3290L.
- Rate: $\frac{dV}{dt} = \frac{3290}{10} = 329L\min^{-1}$

b. Making escape simultaneously with overflow for T=1min.:

- *t = 9 min.* (water reaches top of head)
- ∴Volume when *t = 9 min:* = 329 × 9
  = 2961 L
- h_w at V = 2961 L:

$$h_w(2961) = \frac{4}{7}\left(e^{\frac{7(2961)}{4000\pi}} - 1\right)m.;\{V:0 \le V \le 3290\}$$

$$h_w = 2.4022m$$

∴ h_b = h_w - 1.8
= 2.4022 - 1.8
= 0.60 m

Hence required block height for this situation is 60cm.

c. h_w in terms of *t = t (minutes)* after filling commences:

- $\frac{dV}{dt} = 329L\min^{-1}$
- ∴ *V = 329t* (minutes)
- Substituting for V:

$$\therefore h_w(V) = \frac{4}{7}\left(e^{\frac{7V}{4000\pi}} - 1\right)m;\{V:0 \le V \le 3290\}$$

$$\therefore h_w(t) = \frac{4}{7}\left(e^{\frac{7(329t)}{4000\pi}} - 1\right)m.$$

$$\therefore h_w(t) = \frac{4}{7}\left(e^{\frac{2303t}{4000\pi}} - 1\right)m.;\{t:0 < t \le 10\}$$

d. i. h_w in terms of *T(min).*

- ∴t_r = *(10 - T):* Time (minutes) required to reach top of head before it is submerged for *T(minutes).*

$$\therefore h_w(t) = \frac{4}{7}\left(e^{\frac{2303t}{4000\pi}} - 1\right)$$

$$\therefore h_w(T) = \frac{4}{7}\left(e^{\frac{2303(10-T)}{4000\pi}} - 1\right)$$

∴ Height of block required in terms of submergence time:

$h_b = h_w - 1.8$

$$\therefore h_b = \frac{4}{7}\left(e^{\frac{2303(10-T)}{4000\pi}} - 1\right)m. - 1.8m.$$

$$h_b = \frac{4}{7}\left(e^{\frac{2303(10-T)}{4000\pi}} - 1\right)m. - \frac{18}{10}m.$$

$$h_b = \frac{4}{7}\left(e^{\frac{2303(10-T)}{4000\pi}} - 1 - \frac{63}{20}\right)m.$$

$$h_b = \frac{4}{7}\left(e^{\frac{2303(10-T)}{4000\pi}} - \frac{83}{20}\right)m.$$

$$h_b = \frac{4}{140}\left(20e^{\frac{2303(10-T)}{4000\pi}} - 83\right)m.$$

$$h_b(T) = \frac{1}{35}\left(20e^{\frac{2303(10-T)}{4000\pi}} - 83\right)m.;\left\{T{:}0\,\text{min.} \le T \le \left(10 - \frac{4000\pi}{2303}\log_e 4.15\right)\text{min.}\right\}$$

- Restriction: $T \geq 0min.$

$$h_b \geq 0m.$$

$$0. \le \frac{1}{35}\left(20e^{\frac{2303(10-T)}{4000\pi}} - 83\right)m.$$

$$0 \le 20e^{\frac{2303(10-T)}{4000\pi}} - 83$$

$$83 \le 20e^{\frac{2303(10-T)}{4000\pi}}$$

$$4.15 \le e^{\frac{2303(10-T)}{4000\pi}}$$

$$\log_e(4.15) \le \frac{2303(10-T)}{4000\pi}$$

$$\frac{4000\pi}{2303}\log_e(4.15) \le 10 - T$$

$$T \le 10 - \frac{4000\pi}{2303}\log_e(4.15) \approx 2.23\,\text{min.}$$

$$\therefore \left\{T{:}0\,\text{min.} \le T \le \left(10 - \frac{4000\pi}{2303}\log_e 4.15\right)\text{min.}.\right\}$$

Verification of formula:

- $T = 1\ min;\ h_b = 0.60m$:

$$h_b(T) = \frac{1}{35}\left(20e^{\frac{2303(10-T)}{4000\pi}} - 83\right);\left\{T{:}0 \le T \le 10 - \frac{4000\pi}{2303}\log_e 4.15\right\}$$

$$h_b(1) = \frac{1}{35}\left(20e^{\frac{2303(10-1)}{4000\pi}} - 83\right)$$

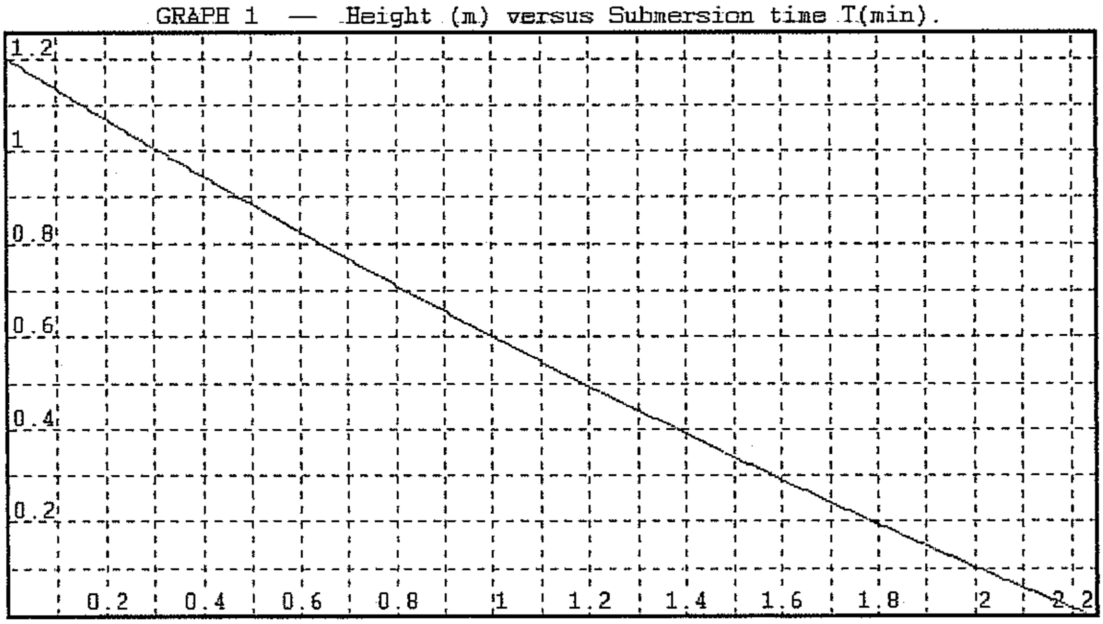
$$h_b = \frac{1}{35}\left(20e^{\frac{20727}{4000\pi}} - 83\right)$$

$$h_b = 0.6022m$$

$$h_b = 0.60m$$

d. ii. Graph of h_b versus T(min):

GRAPH 1 — Height (m) versus Submersion time T(min).



- *As -T* indicates, graph experiences exponential decay ∴ graph is reflected in Y-Axis.
- Does not display asymptotic behaviour, as outside of restricted domain.
- Maximum height occurs when *t=0min*. as Matthew and block are level with flask top.
- Maximum *T (2.231min.)* when $h_b = 0m$.

(Question 5): Volume of Matthew and block considered:

- Block: r_b = *13cm; h_b = 0.60m.*
- Matthew; r_m = *13: h_m = 1.8m.*

For h_b = *0.60m*, volume ***(V$_c$)L.*** required to just cover Matthew's head:

- ***Let Volume of block & Matthew = $V_{(b+m)}$***

$$V_{(b+m)} = (\pi r_b^2\, h_b + \pi r_m^2\, h_m)m.^3 \times 1000$$
$$= 1000(\pi(0.13)^2 x0.6 + \pi(0.13)^2 x1.80))\, L.$$
$$= 1000(0.0169\pi \times 0.60 + 0.0169\pi \times 1.80)$$
$$=1000(0.01014\pi + 0.03042\pi)$$
$$=10.14\pi + 30.42\pi$$
$$= 40.56\pi L.$$

$\therefore$ height of Matthew and block is 2.4m *(+1m. due to stage)*, $\therefore$ ***h_f*** = *3.4m* Hence the volume of water required is:

- ***$\therefore$ V_c = maximum volume V(h) - $V_{(b+m)}$***

$$V_c = V(h) - V_{(b+m)}$$
$$V_c = \frac{4000\pi}{7}\left(\log_e \frac{(7h-3)}{4}\right)L. - 40.56\pi L.$$
$$V_c = \frac{4000\pi}{7}\left(\log_e \frac{(7 \times 3.4 - 3)}{4}\right)L. - 40.56\pi L.$$
$$V_c = \frac{4000\pi}{7}(\log_e 5.2)L. - 40.56\pi L.$$
$$V_c = 2959.66L. - 40.56\pi L.$$
$$V_c = 2832.238L.$$
$$V_c = 2832L.$$

b. i. *V_c expressed in H (Block height in cm).*

Let:

- ***H = Block height (cm)***
- ***h_m = 180cm.***

$$V_{(b+m)} = \left(\pi r_b^2 H + \pi r_m^2 h_m\right)m^3$$
$$V_{(b+m)} = \left(\pi r^2 H + \pi r^2 \times 180\right) \times 1000L.$$
$$V_{(b+m)} = \pi(0.13)^2 \times \frac{(H+180)}{100} \times 1000L.$$
$$V_{(b+m)} = 16.9\pi \frac{(H+180)}{100} L.$$

In terms of H(cm); $h_f = \left(\frac{H+180}{100}\right) + 1m$

∴ Volume in H(cm):

$$V_c = \frac{4000\pi}{7}\left(\log_e \frac{\left(7\left(\left(\frac{H+180}{100}\right)+1\right)-3\right)}{4}\right) - 16.9\frac{H+180}{100}\pi L.$$

$$V_c = \frac{4000\pi}{7}\left(\log_e \frac{7\left(\left(\frac{(H+180)}{100}+\frac{100}{100}\right)-\frac{300}{100}\right)}{4}\right) - 16.9\frac{H+180}{100}\pi L.$$

$$V_c = \frac{4000\pi}{7}\left(\log_e \frac{\frac{7H+1260}{100}+\frac{700}{100}-\frac{300}{100}}{4}\right) - 16.9\frac{H+180}{100}\pi L.$$

$$V_c = \frac{4000\pi}{7}\left(\log_e \frac{\frac{7H+1660}{100}}{4}\right) - 16.9\frac{H+180}{100}\pi L.$$

$$V_c = \frac{4000\pi}{7}\left(\log_e \frac{7H+1660}{400}\right) - 0.169\pi(H+180)L.;\{H{:}0cm.\leq H \leq 120cm.\}$$

Verification: When $H = 60cm.$; $V_c = 2832L.$

$$V_c = \frac{4000\pi}{7}\left(\log_e \frac{7H+1660}{400}\right) - 0.169\pi(H+180)L.$$

$$2832 = \frac{4000\pi}{7}\left(\log_e \frac{7(60)+1660}{400}\right) - 0.169\pi(60+180)L$$

$$2832 = \frac{4000\pi}{7}(\log_e 5.2) - 40.56\pi L.$$

$$2832 = 2989.6650 - 40.56\pi L.$$

$$2832 = 2832.24L. \therefore Q.E.D.$$

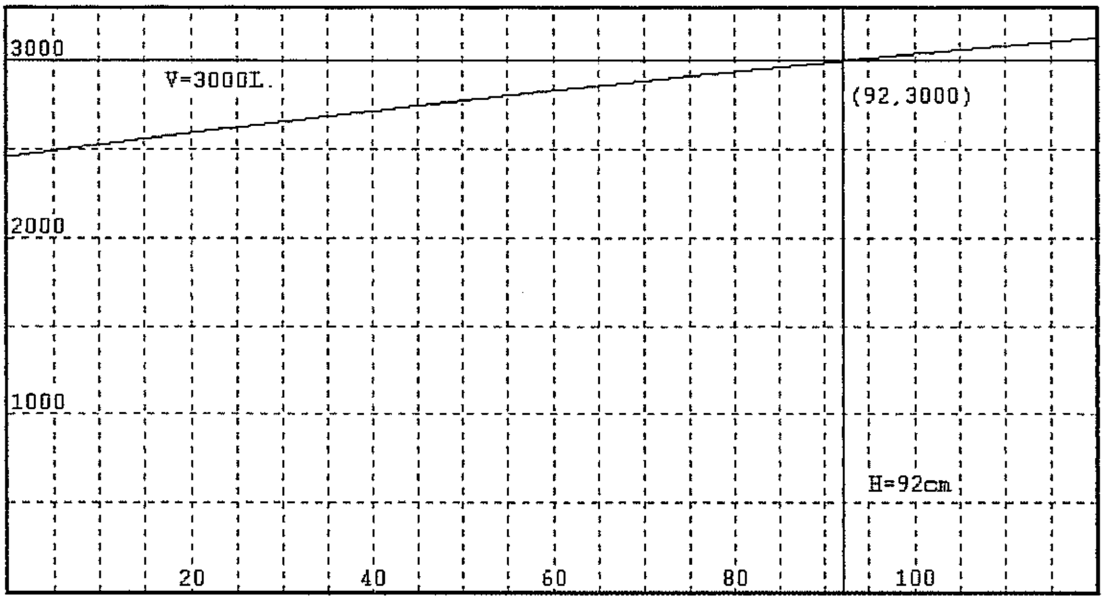
b. ii. When rate of water entering flask = 300L/min. the height so that Matthew's head is just covered after 10 minutes:

∴ after 10min. $V_c = 3000L$:

$$V_c = \frac{4000\pi}{7}\left(\log_e \frac{7H+1660}{400}\right) - 0.169\pi(H+180)L.$$

$$3000 = \frac{4000\pi}{7}\left(\log_e \frac{7H+1660}{400}\right) - 0.169\pi(H+180)L.$$

To find H, the height was read of a graph where $V = 3000L$ and $V(H)$ equations intersect and verified using spreadsheet; (APPENDIX 1):



$\therefore$ Block height = 92cm.

(Question 6):

a.

- Radius of block = R(cm)
  Taken: Not < 13cm. As was previously used and $\therefore$ acceptable to Matthew and no larger than 100cm.

- Volume of Matthew: 30.42πL
- Volume of block ***V_b(L.)***:

$$V_b = \pi r^2 h$$

$$V_b = \pi\left(\left(\frac{R}{100}\right)^2 \times \frac{H}{100}\right) m^3 \times 1000$$

$$V_b = \pi\left(\frac{R^2}{10000} \times \frac{H}{100}\right) \times 1000L.$$

$$V_b = 0.001\pi R^2 H(L.)$$

V_c expressed in H(cm) & R(cm):

$$V_c = \frac{4000\pi}{7}\left(\log_e \frac{7H + 1660}{400}\right) - 0.001\pi R^2 H - 30.42\pi L.$$

$\{H: 0cm \leq H \leq 120cm\} \cap \{R: 13cm \leq R < 100cm\}$

- Verification: When $H = 60cm$; $R = 13cm$; $V_c = 2832L$.

$$V_c = \frac{4000\pi}{7}\left(\log_e \frac{7H + 1660}{400}\right) - 0.001\pi R^2 H - 30.42\pi L.$$

$$2832 = \frac{4000\pi}{7}\left(\log_e \frac{7(60) + 1660}{400}\right) - 0.001\pi(13)^2(60) - 30.42\pi L.$$

$$2832 = \frac{4000\pi}{7}(\log_e 5.2) - 10.14\pi - 30.42\pi L.$$

$$2832 = 2959.6654 - 40.56\pi L.$$

$$2832 = 2832.24L.$$

$$2832 = 2832L.$$

b. Set of R, H and F to cover Matthew's head in 10 minutes:

- **Let F = flow rate.**
  As $V_c = F.t$; where t = 10min;
  $\therefore V_c = 10.F$;

$\therefore$ General formula of F found by substituting for V_c:

$$V_c = \frac{4000\pi}{7}\left(\log_e \frac{7H + 1660}{400}\right) - 0.001\pi R^2 H - 30.42\pi L.$$

$$10F = \frac{4000\pi}{7}\left(\log_e \frac{7H + 1660}{400}\right) - 0.001\pi R^2 H - 30.42\pi L.$$

$$F = \frac{400\pi}{7}\left(\log_e \frac{7H + 1660}{400}\right) - 0.0001\pi R^2 H - 3.042\pi L / \text{min}.$$

- Initially Flow rate range: ***{F:180L/min. < F < 330L/min.}***

RESTRICTIONS:

If concrete block was lowered into the flask, it's maximum radius would be 0.4m. However, I have assumed that flask is constructed over block and hence dimensions of the block are restricted by the dimensions of flask. For a given block radius R(cm), a maximum height H(cm) for which block will contact with inner walls applies **(See Fig. 2 below)**:

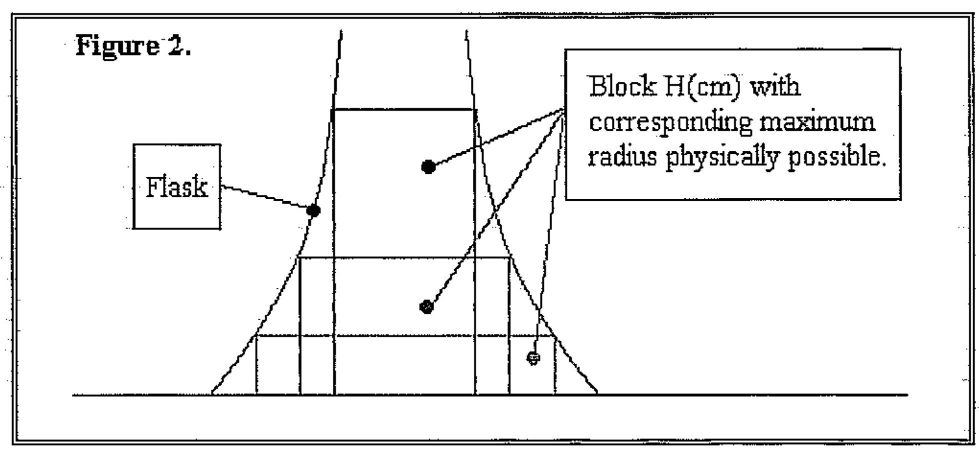


Figure 2.

- Where ***x* = *r* (m)** ; ***y* = height (m)** from floor. ∴ expressed in **R(cm)** and **H (cm)** giving maximum H(cm) for R(cm):

General equation:

$$y = \frac{4}{7x^2} + \frac{3}{7}(m.)$$

$$\frac{H}{100} + 1 = \frac{4}{7\left(\frac{R}{100}\right)^2} + \frac{3}{7}(cm.)$$

$$\frac{H}{100} = \frac{4}{\frac{7R^2}{10000}} + \frac{3}{7} - \frac{7}{7}(cm.)$$

$$\frac{H}{100} = \frac{4}{\frac{7R^2}{10000}} - \frac{4}{7}(cm.)$$

$$\frac{H}{100} = \frac{40{,}000}{7R^2} - \frac{4}{7}(cm.)$$

$$H(R) = \frac{400}{7}\left(\frac{10{,}000}{R^2} - 1\right)(cm.)$$

Verification:

When *R = 40 cm; H = 300cm*

$$H = \frac{400}{7}\left(\frac{10{,}000}{R^2} - 1\right)$$

$$H = \frac{400}{7}\left(\frac{10{,}000}{40^2} - 1\right)$$

$$H = \frac{400}{7}\left(\frac{10{,}000}{1600} - 1\right)$$

$$H = \frac{400}{7}(6.25 - 1)$$

$$H = \frac{400}{7} \times 5.25$$

$$H = 300cm.$$

Height of block also restricted to a maximum height of 120cm so as Matthew's head remains no higher than tank top; and minimum of 0cm.

∴ 0cm ≤ H ≤ 120cm

Radius has been previously set at 13cm, assuming this is the minimum radius required for Matthew to stand safely on block:

∴ 13cm > *R* > 100cm (*R* is not > 100cm due to tank dimensions)

For a given height, the maximum radius determined by:

$$H = \frac{400}{7}\left(\frac{10{,}000}{R^2} - 1\right)(cm.)$$

$$\frac{7H}{400} = \frac{10{,}000}{R^2} - 1$$

$$\frac{7H}{400} + \frac{400}{400} = \frac{10{,}000}{R^2}$$

$$\frac{7H + 400}{400} = \frac{10{,}000}{R^2}$$

$$\frac{400}{7H + 400} = \frac{R^2}{10{,}000}$$

$$R = \sqrt{10{,}000 \times \left(\frac{400}{7H + 400}\right)}$$

$$R = \sqrt{\frac{4{,}000{,}000}{7H + 400}}cm.$$

Verification:

When *R = 40 cm; H = 300cm*

$$R = \sqrt{\frac{4{,}000{,}000}{7(300) + 400}}(cm.)$$

$$R = \sqrt{\frac{4{,}000{,}000}{2100 + 400}}$$

$$R = \sqrt{\frac{4{,}000{,}000}{2500}}$$

$$R = \sqrt{1600}$$

$$R = 40cm.$$

Maximum flow rate (F) will occur for largest volume to fill for given time of 10 minutes. ∴ Maximum flow rate will be given by R = 13cm. (unrestricted by previous equations).

$$.F = \frac{400\pi}{7}\left(\log_e \frac{7H+1660}{400}\right) - 0.0001\pi R^2 H - 3.042\pi L \text{ / min.}$$

$$.F = \frac{400\pi}{7}\left(\log_e \frac{7H+1660}{400}\right) - 0.0001\pi(13)^2 H - 3.042\pi L \text{ / min.}$$

$$.F = \frac{400\pi}{7}\left(\log_e \frac{7H+1660}{400}\right) - 0.0169\pi H - 3.042\pi L \text{ / min.}$$

∴ F(L/min.) versus H(cm) {H: 0cm ≤ H ≤ 120cm} is graphed to give the upper limit of flow rate for a particular height:

GRAPH 2. – Curve (A)

Minimum (F) will occur for least volume to fill, ∴ for a particular height block will occupy maximum volume when matched with it's maximum radius.

$$R = \sqrt{\frac{4{,}000{,}000}{7H+400}} cm.$$

$$\therefore R^2 = \frac{4{,}000{,}000}{7H+400} cm$$

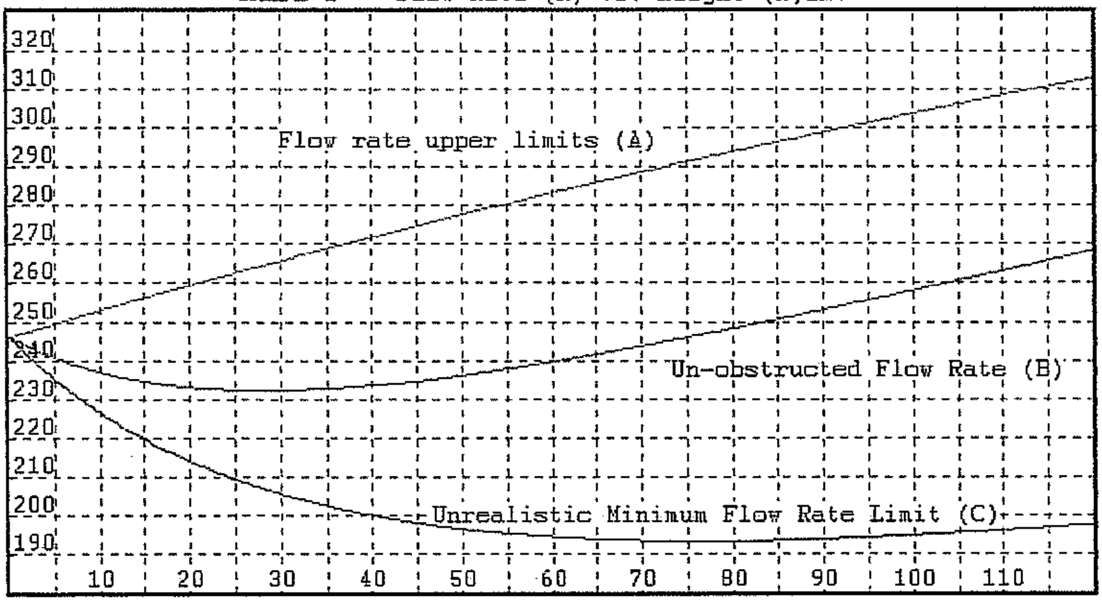
Flow rate in terms of H(cm) will give lower limit on flow rate by giving block height with it's maximum possible radius:

$$.F = \frac{400\pi}{7}\left(\log_e \frac{7H+1660}{400}\right) - 0.0001\pi R^2 H - 3.042\pi L \text{ / min.}$$

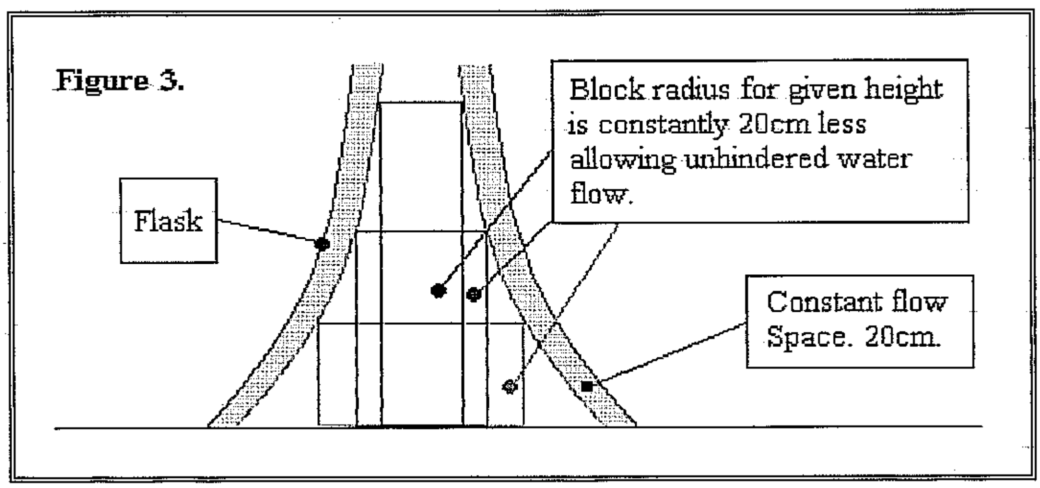
$$F = \frac{400\pi}{7}\left(\log_e \frac{7H+1660}{400}\right) - 0.0001\pi\left(\frac{4{,}000{,}000}{7H+400}\right) H - 3.042\pi L \text{ / min..}$$

GRAPH 2. – Curve (C)

GRAPH 2 -- Flow Rate (R) vs. Height (H)cm.



Curve (C) gives minimum *(F)L/min.* however, contacting with the walls completely stops passage of water. Hence for a given height, the corresponding radius is reduced by a constant 20cm and is assumed will fully negate retardation on the flow rate. **(See Fig. 3 Below):**



∴ Lower limit of flow rate for particular height is now given as:

- ***(R - 20)cm; Let this = R_u***

$$R = \sqrt{\frac{4{,}000{,}000}{7H+400}}\,cm.$$

$$\therefore R_u = \sqrt{\frac{4{,}000{,}000}{7H+400}} - 20cm$$

$$R_u^2 = \left(\sqrt{\frac{4{,}000{,}000}{7H+400}} - 20\right)^2 cm$$

$$\therefore F = \frac{400\pi}{7}\left(\log_e \frac{7H + 1660}{400}\right) - 0.0001\pi R^2 H - 3.042\pi L \, / \, \text{min.}$$

$$F = \frac{400\pi}{7}\left(\log_e \frac{7H + 1660}{400}\right) - 0.0001\pi\left(\sqrt{\frac{4{,}000{,}000}{7H + 400}} - 20\right)^2 H - 3.042\pi L \, / \, \text{min..}$$

$\therefore$ for a given radius (cm), the maximum height (cm) so as to make contact with wall is given. As contacting the wall will prevent water flowing up past block (water assumed as entering at bottom of tank), the maximum height will actually be smaller than what is given. Hence:

$$H < \frac{400}{7}\left(\frac{10{,}000}{r^2} - 1\right)$$