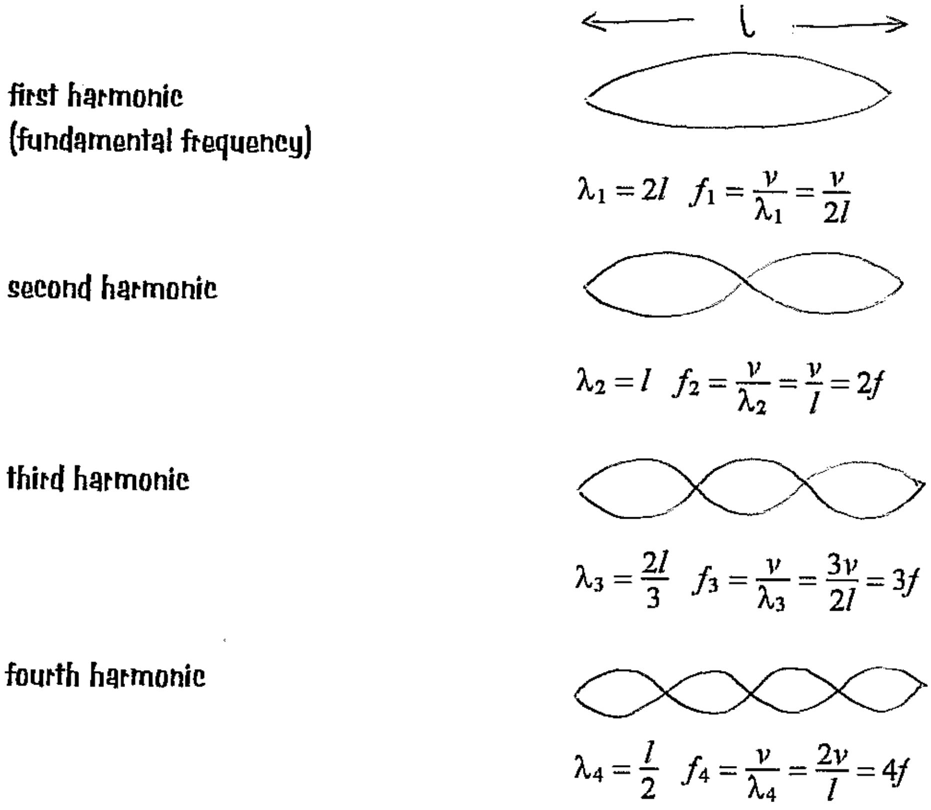
Article ...

The vibrating system of violin is basically a string fixed at both ends, which is excited by bowing or plucking. When plucked or bowed, it vibrates at more than one frequency. Since nodes are produced at both ends, the possible wavelengths are determined by the length of the string. The lowest frequency produced (fundamental frequency) is where wavelength is twice the length of string. This harmonic is called f_1. The other harmonics, f_2, f_3 ...... f_n have frequencies according to the rule $f_n = n\,f$. [1](see figure 1) The number of harmonics produced can be influenced by the point at which string is made to vibrate. If the string is plucked at midpoint, a stationary point cannot exist there, therefore even harmonics will not be produced.

Figure 1: Harmonics on a string



---

[1] p. 29, Armitage, F., et al. Physics Two. Heinemann Educational Australia, Port Melbourne, 1991.

The frequencies can also be described in the form of: $f_n = \frac{n}{2l}\sqrt{\frac{T}{\pi r^2 \rho}}$ [2]

where		
	n	1, 2, 3 ...... (harmonic No.)
	l	length of string
	r	radius of string
	p	density of the material from which it is made
	T	its tension

From the formula above, the **Laws of Stretched Strings** were derived:

- The vibration frequency of a stretched string is inversely proportional to its length.
- It is directly proportional to the square root of the tension.
- It is inversely proportional to the square root of the mass per unit length ($\pi r^2 \rho$). [3]

These laws are common knowledge to string players. Pressing the string against fingerboard shortens it and raises the pitch. This is the first law. To change the tuning, pitch is raised by tightening the adjusting screws - the second law. And the lower strings are always thicker than the higher ones, in accordance with the third law.

On a violin, sound is "maintained" by driving force when bowing. When plucking, its sound is "transient" which dies away after an initial driving force. When bowing, players continue to supply energy to instrument in an amount just sufficient to make up the losses from friction, sound radiations, etc. so the amplitude of vibrations remains constant and sound is steady. In the transient case a rather large amount of energy is supplied by the plucking action and sets up a vibration of large amplitude. Then, since no more energy is supplied, the amplitude of the vibration, and with it the sound intensity, gradually decays as energy is lost by friction and sound radiation. [4]

In this investigation, relationship between the frequency of the note and the time needed for the sound plucked on the violin string to die away is explored. Since on a violin, the envelope of the sound is generally constant, the timbre and quality of the sound produced will be discussed in relation to thickness and material of the string.

---

[2] p. 11, Fletcher, N. Physics And Music. Heinemann Educational Australia, Richmond, 1976.

[3] p. 71, Johnston, I. Measured Tones, The Interplay Of Physics And Music. Adam Hilger, Bristol and New York, 1989.

[4] P. 8, Fletcher, N. Physics And Music. Heinemann Educational Australia, Richmond, 1976.

Extended Practical Investigation

Plucked Strings

Purpose ...

To investigate the decay of string sound by:

- varying the frequencies played on one string.
- playing the same frequency on strings with different material or thickness

To compare each string of various frequencies, material or thickness with its timbre.

To support the following hypothesis: [1]

- The normal frequency of vibration of a stretched string is inversely proportional to its length.

Variables ...

1. Variables tested:

- pitch (G, D, A, E). The strings have different density and tension, different frequency from each string is expected.
- frequency of each string. It is varied by progressively shorten one string at a time. The effect of this variable is the subject of the investigation.
- material of strings (silver, aluminium, steel). The material used may influence the density of strings.
- thickness. The thickness of string is related to density. If the same note is played on strings with different thickness, it may be explored whether if diameters of strings contribute to the qualities of sounds produced.

---

[1] Johnston, I. Measured Tones, The Interplay Of Physics And Music, Adam Hilger, Gristol and New York, 1989.

2. Variables kept constant:

- violin. The violin would amplify the vibrations produced by strings, therefore this variable would be expected to have the greatest effect on the sound quality.
- brand of strings. This ensures the products are of same quality.
- room and position where tests were taken. Resonance of the room may affect certain harmonics of the strings.
- frequency response of microphone, computer program used. This would distort the quality of sound recorded.
- distance between vibrating strings and microphone. Varying the distance may affect volume recorded hence the accuracy of experiment.
- position of violin being held. Holding main body (sound box) of violin was avoided as it may affect whole vibration system.
- force used for plucking. There may have been slight variation in the amount of force applied in each pluck due to uncontrollable human errors (although it did come with practise). If it was interfered in any manner, the measurement was discarded from result.

3. Uncontrollable variables:

- temperature and humidity. The instability of these two factors may affect the characteristics of strings. The temperature and humidity may cause slight change in volume and density of the strings. To minimise the errors, strings were tuned using chromatic tuner after every pluck to ensure the frequency required was present.

Materials ...

For maximum accuracy, all equipment used were the best available for each part of the set-up.

1. Violin: made in Budapest by Andras Bergmann, 1910 with two-piece maple back, maple neck and side.

2. Microphone: Uni-directional dynamic microphone (Shang-hung Model 807).

3. Analysis Equipment:

   - IBM compatible 486dx2-66

   - Software: Demonstration version of "Cool Edit Version 1.31" by David Johnston (1993). Mono channel and sampling rate at 48kHz with 16 bits resolution.

   - Sound level meter (TECKNOS Model SLM-120): A dBA meter was used to check the consistency of sound intensity level when plucking (around 80 dB).

- Micon chromatic tuner (ARION Model HU-8400): This assisted the tuning of strings to the required frequency and pitch.

- pencil is used to give string a rigid edge when shortening.

- Strings. Violin strings can be metal, pig gut, or gut wound with either silver or aluminium wire.

The following table shows strings of different materials, thicknesses and prices tested in the experiment.

Manufacturer: **Pirastro**
Model: **Synoxa** **(synthetic gut)**

	Material	**Diameter**	**Price**
G string	gut wound with Silver	0.80 mm	$ 21.00
D string	gut wound with Aluminium	0.77 mm	$ 14.00
A string	gut wound with Aluminium	0.63 mm	$ 12.00
E string	steel wound with Aluminium	0.28 mm	$ 8.00

Methods ...

1. In a quiet room, holding microphone 5 cm away, a violin string was plucked of full length, sound was digitised and analysed on computer and a graph of sound wave with amplitude vs time was produced.

2. The time sound takes to decay until no longer audible (about 15 dB, back ground noise level) was measured to the nearest milliseconds by the software used. The same experiment was repeated 5 more times.

3. The string was progressively shortened raising the pitch to the fifth (an increase of a factor 1.5 in frequency) then to the octave (2 ×) the twelfth (3 ×) and fifteenth (two octave or 4 ×) using a pencil and results of the revebration time for each sound were recorded.

4. Same procedures were repeated on other strings.

Figure 1 shows the setting up of experiment:

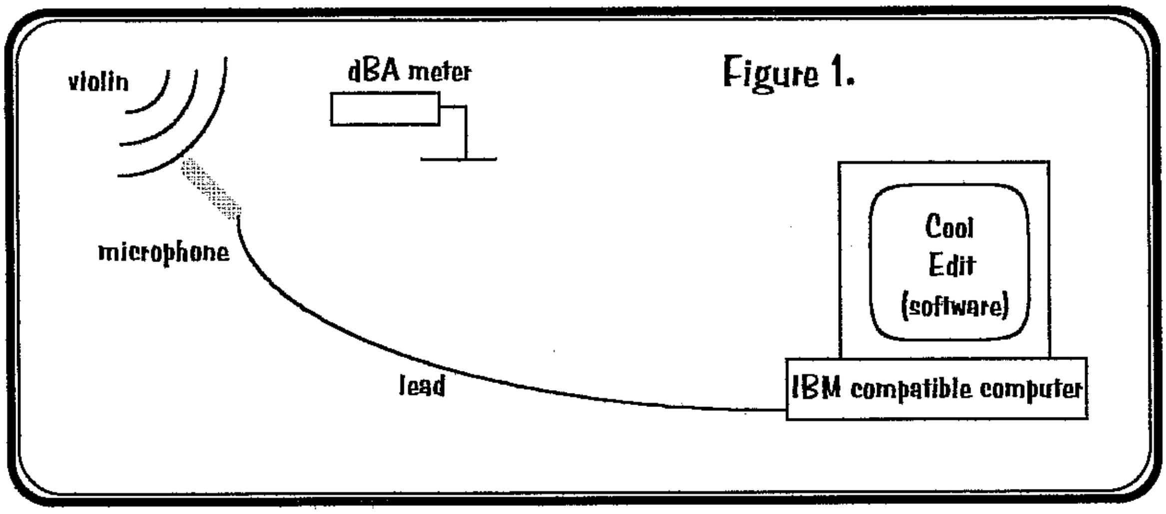


Figure 1.

Strategies Employed To Overcome Difficulties ...

1. Resonance of other strings:
   All of other strings were held while sample was being taken.

2. Background noise:
   A unidirectional dynamic microphone was used and held close to violin (5 cm away) during experiment to minimise unwanted noises.

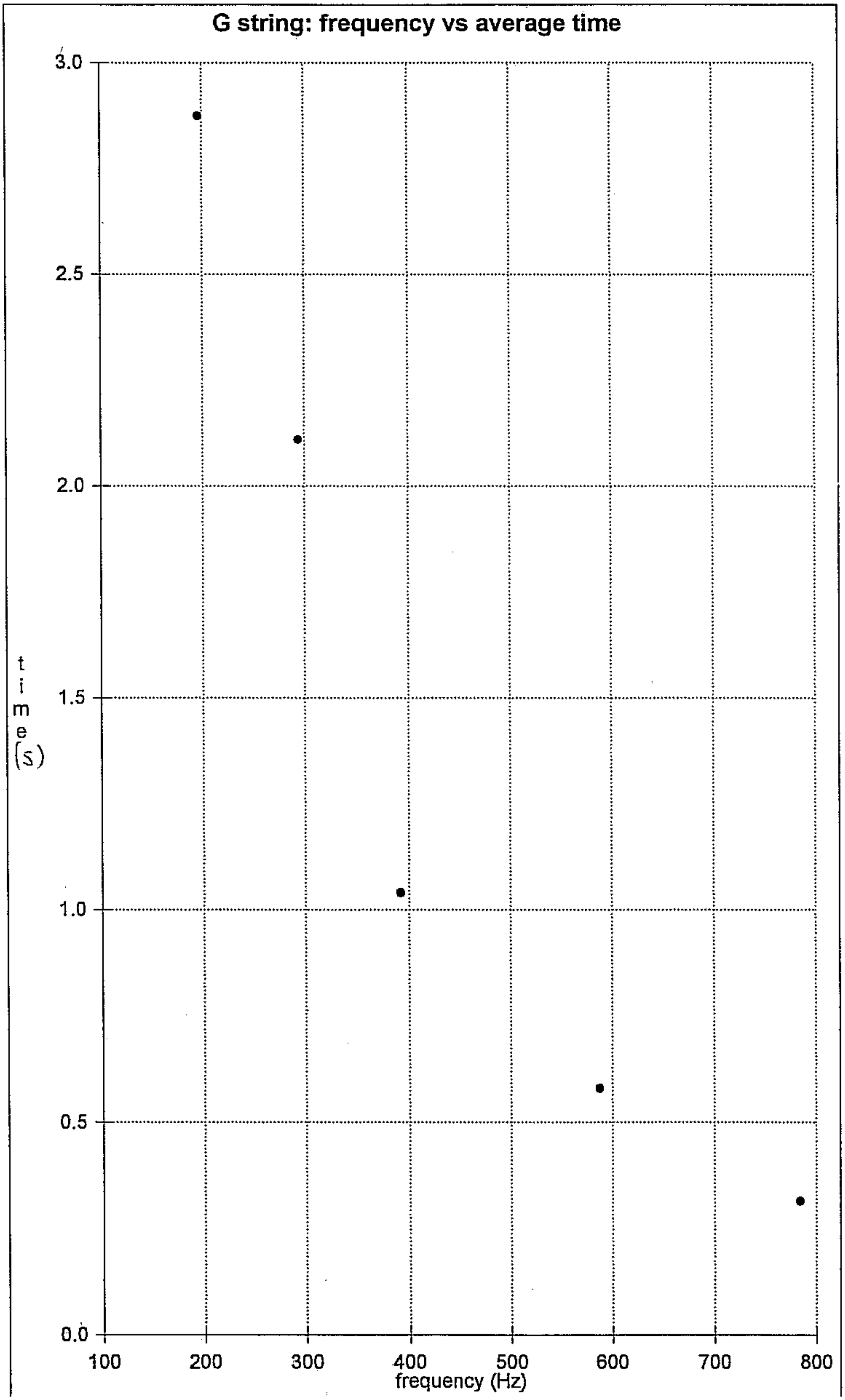
3. Rigid edge of string:
   A hard edge like a pencil was used instead of a soft finger tip which causes damping effect.

4. Resonance of other objects:
   Tests were performed in the same room of the house with all the equipment in same position.

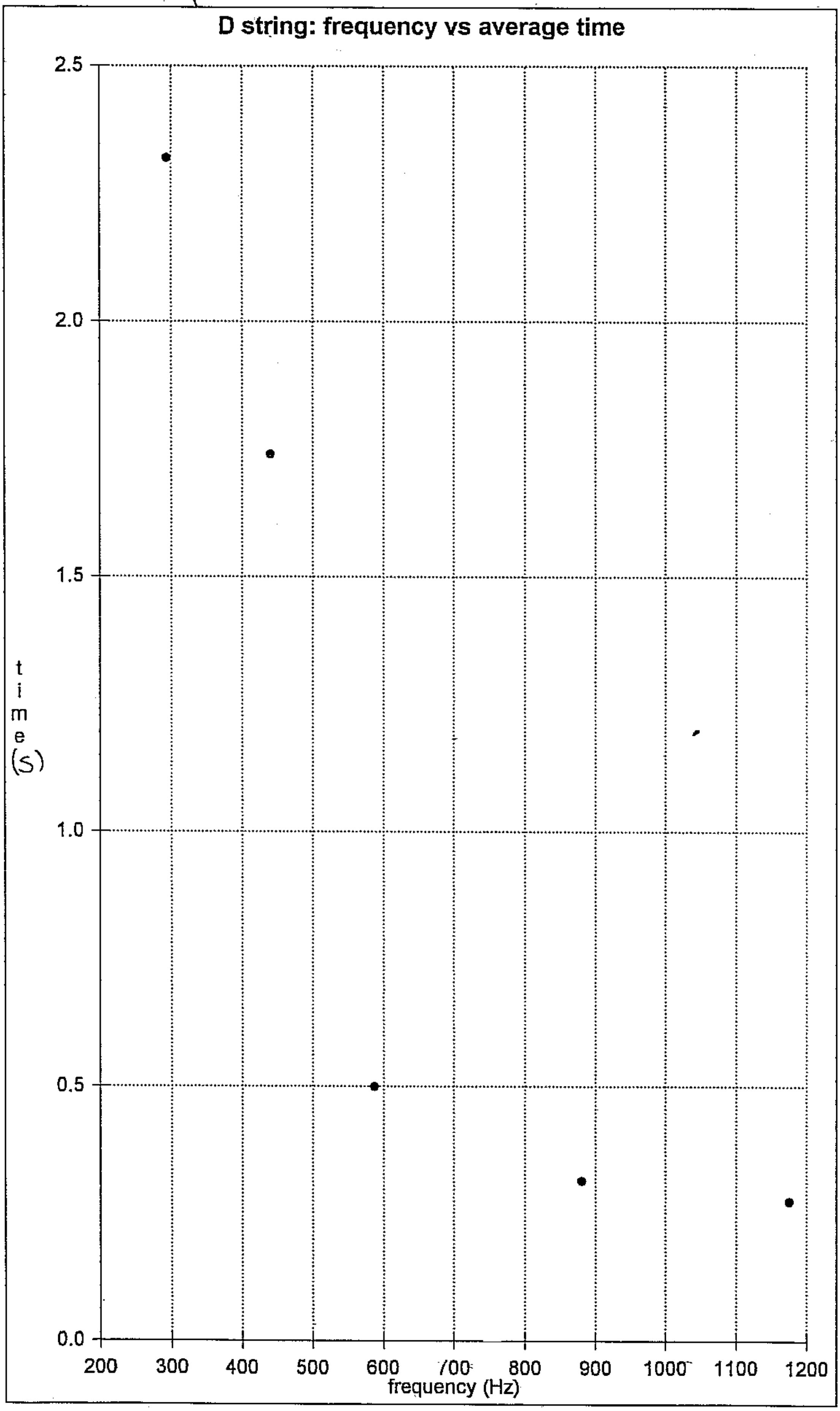
Results ...

According to specifications on equipment and documentation, the error for all graphs are ± 0.02 seconds, ± 2 Hz and ±0.05 cm. For this reason error bars have not been shown. (See Appendix 1 and 2 for table of values)

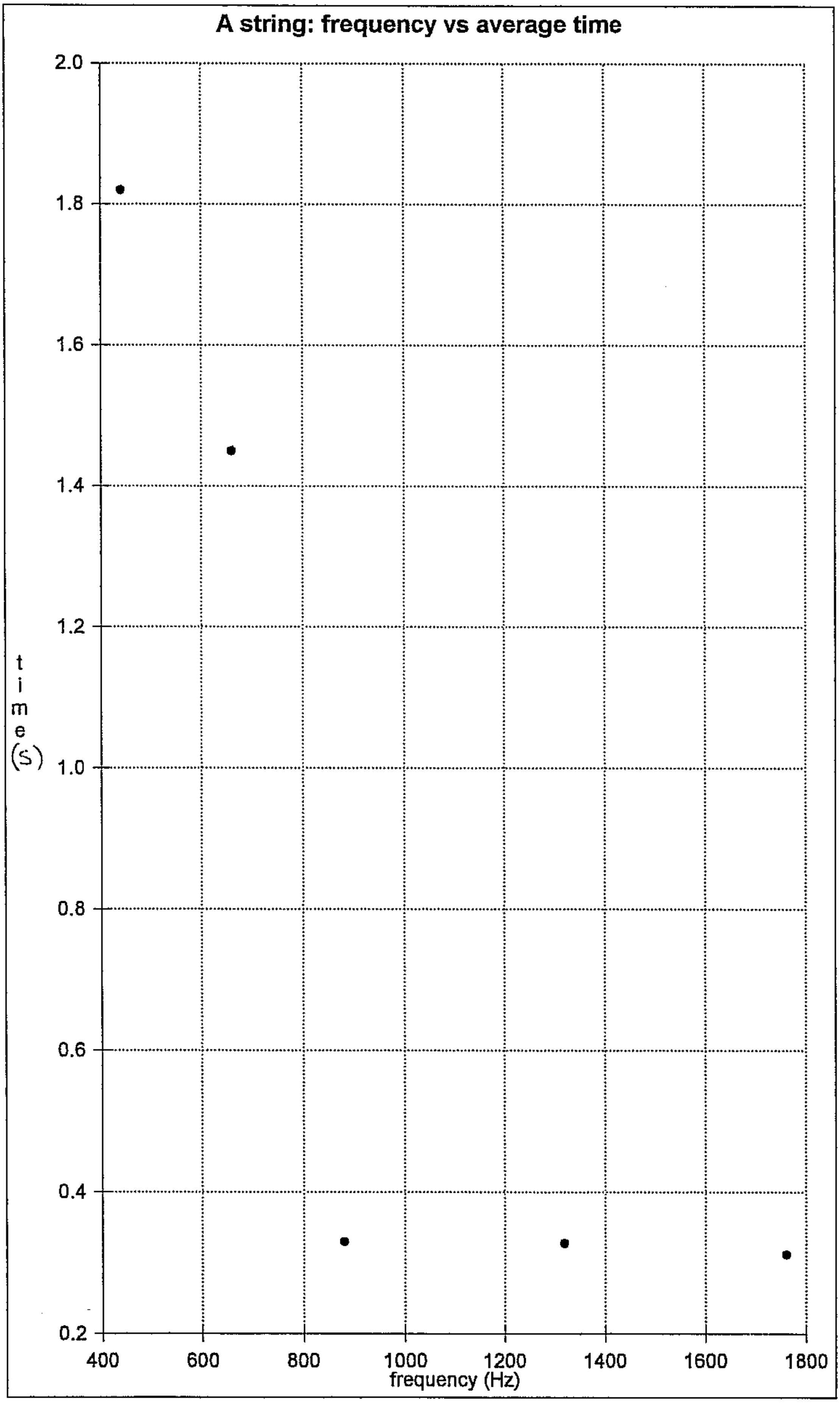
Graph 1


G string: frequency vs average time
time (s)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
100
200
300
400
500
600
700
800
frequency (Hz)

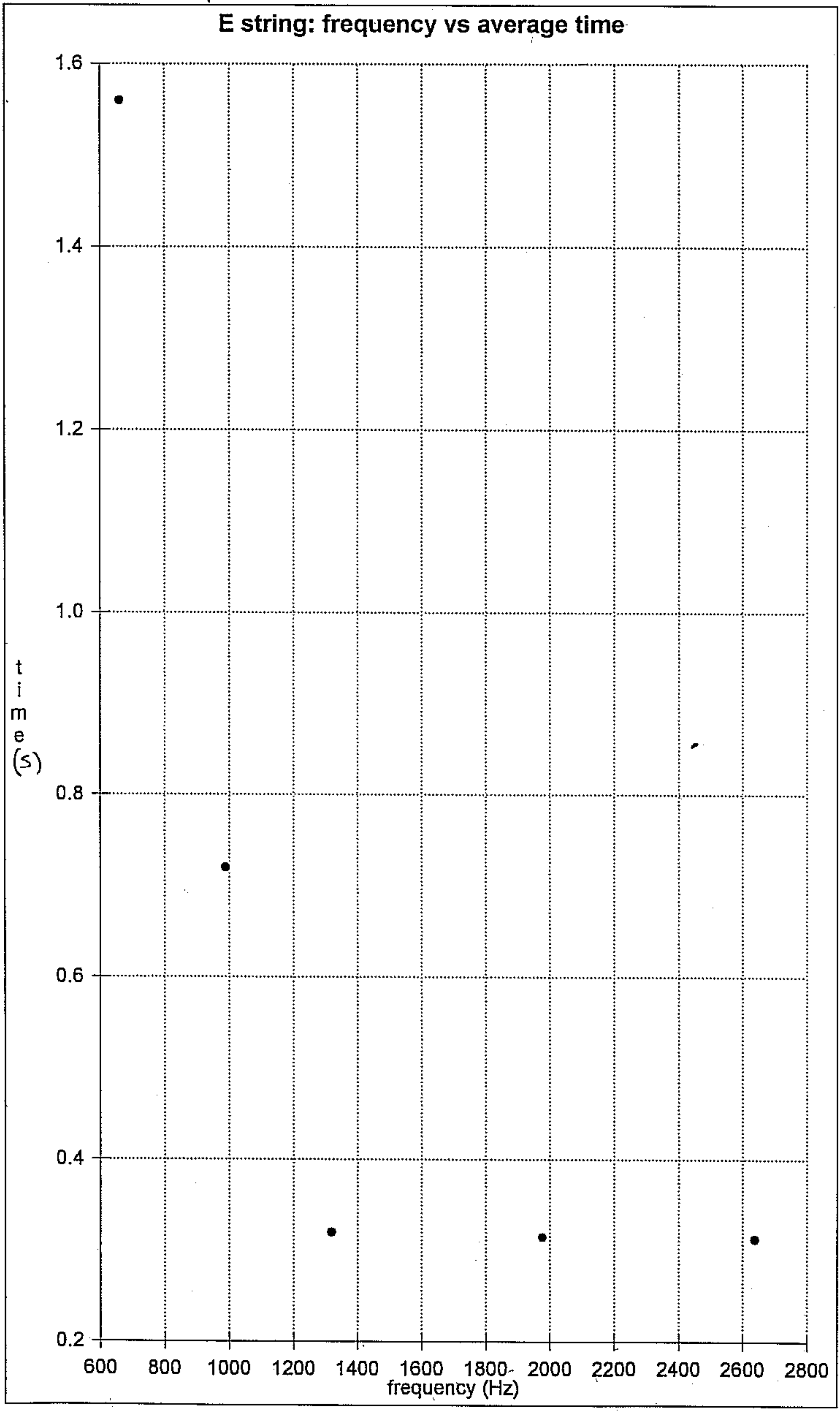
Graph 2


D string: frequency vs average time
2.5
2.0
1.5
1.0
0.5
0.0
time (s)
200
300
400
500
600
700
800
900
1000
1100
1200
frequency (Hz)

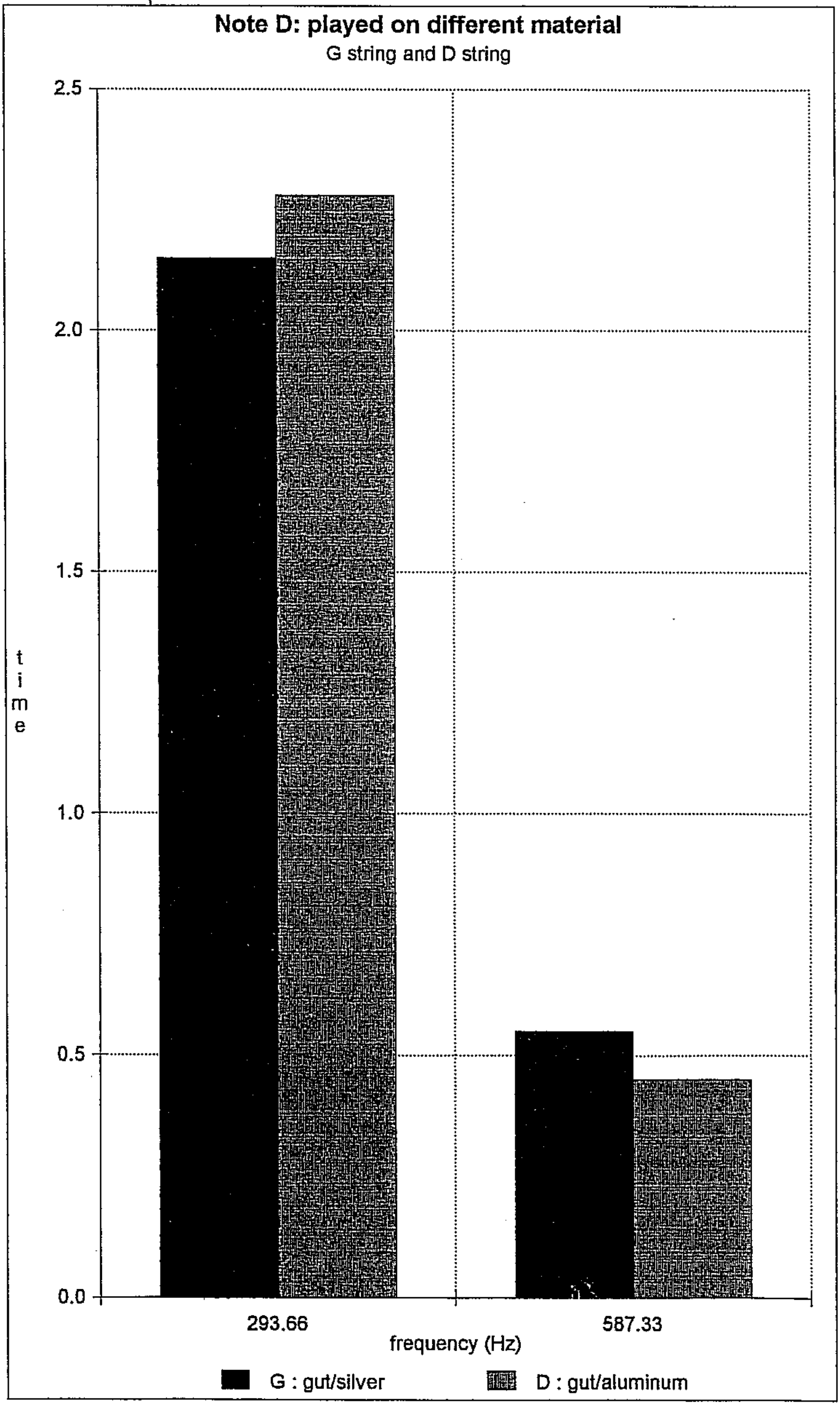
Graph 3


A string: frequency vs average time
time (s)
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
400
600
800
1000
1200
1400
1600
1800
frequency (Hz)

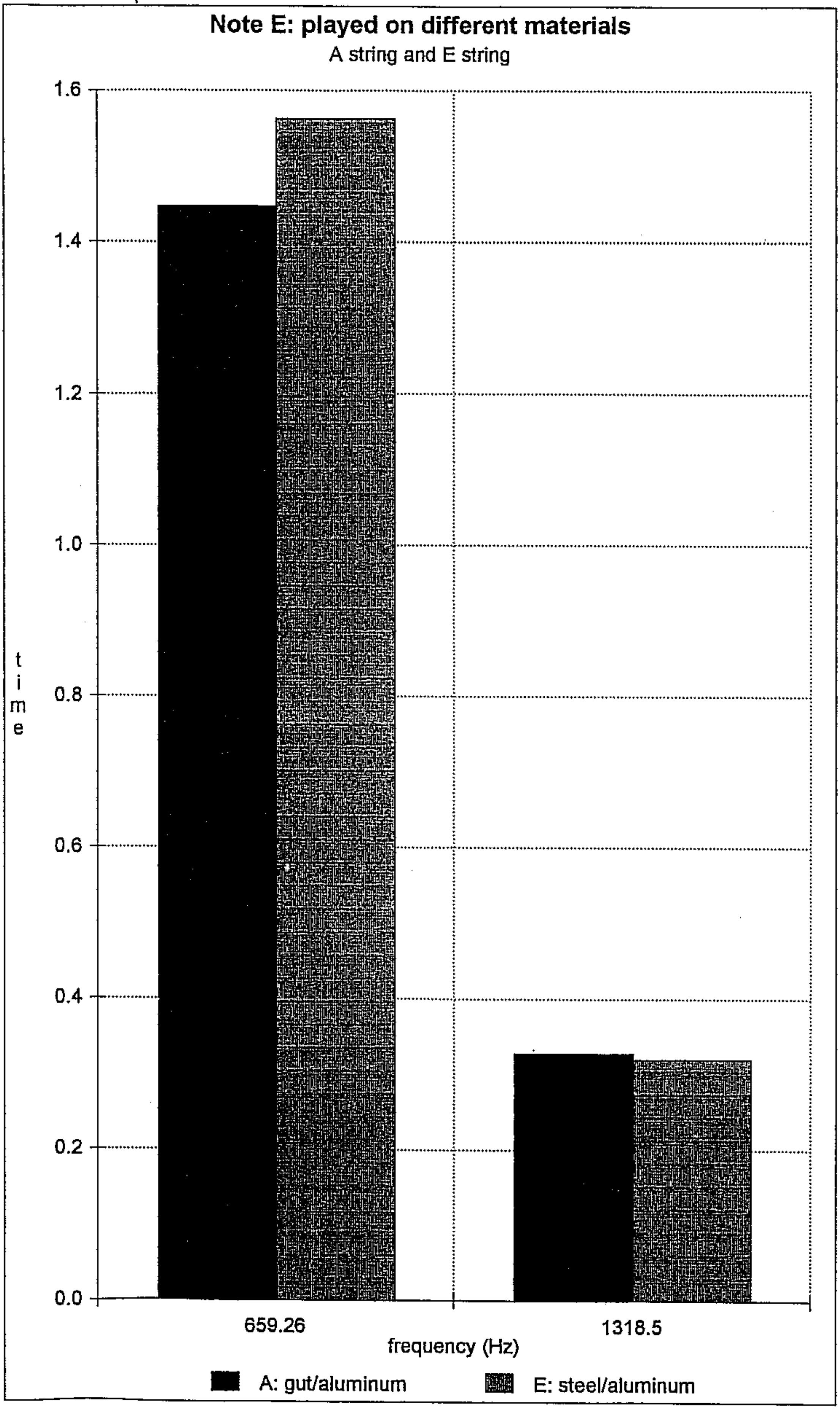
Graph 4


E string: frequency vs average time
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
time (s)
600
800
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
frequency (Hz)

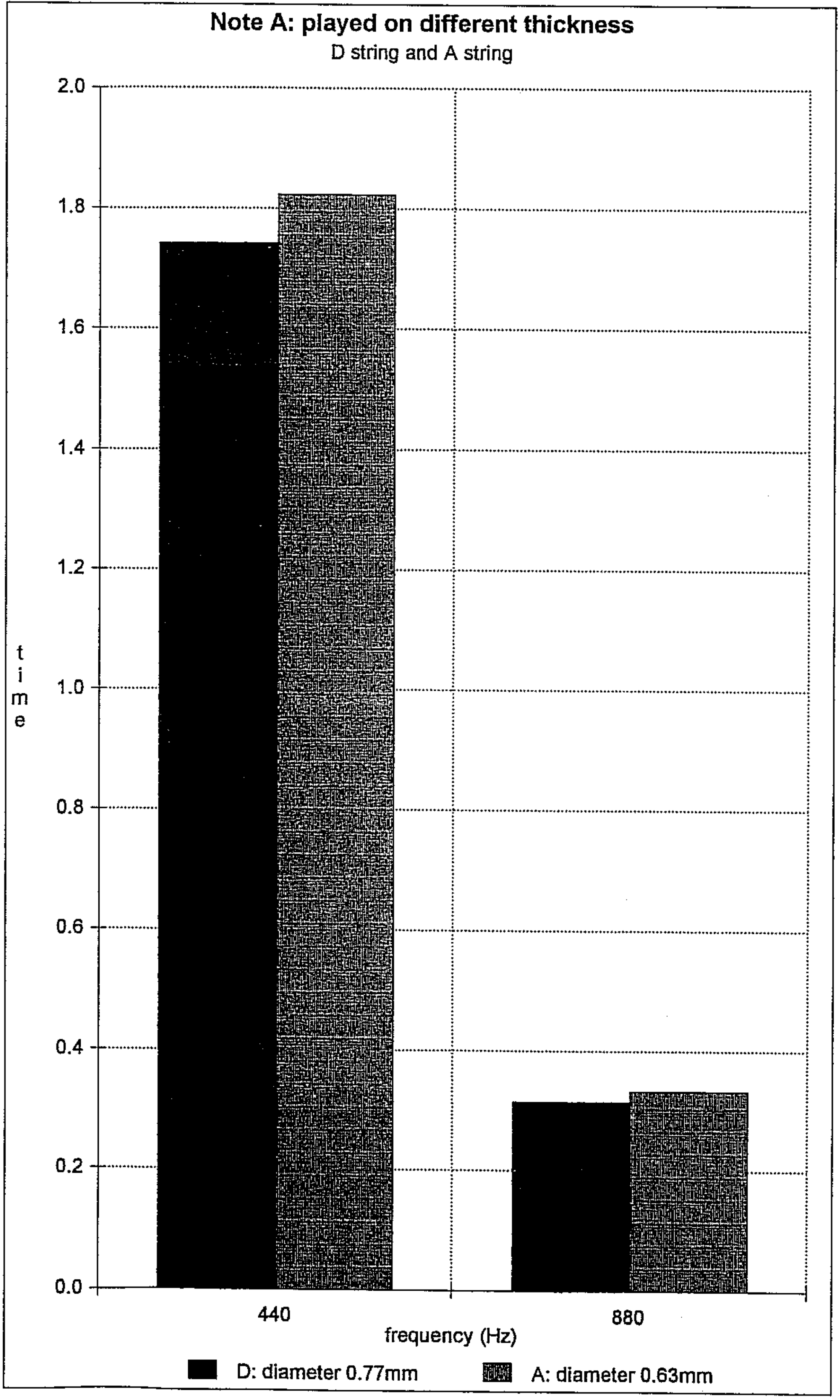
Graph 5


Note D: played on different material
G string and D string
2.5
2.0
1.5
1.0
0.5
0.0
time
293.66
587.33
frequency (Hz)
■ G : gut/silver
▨ D : gut/aluminum

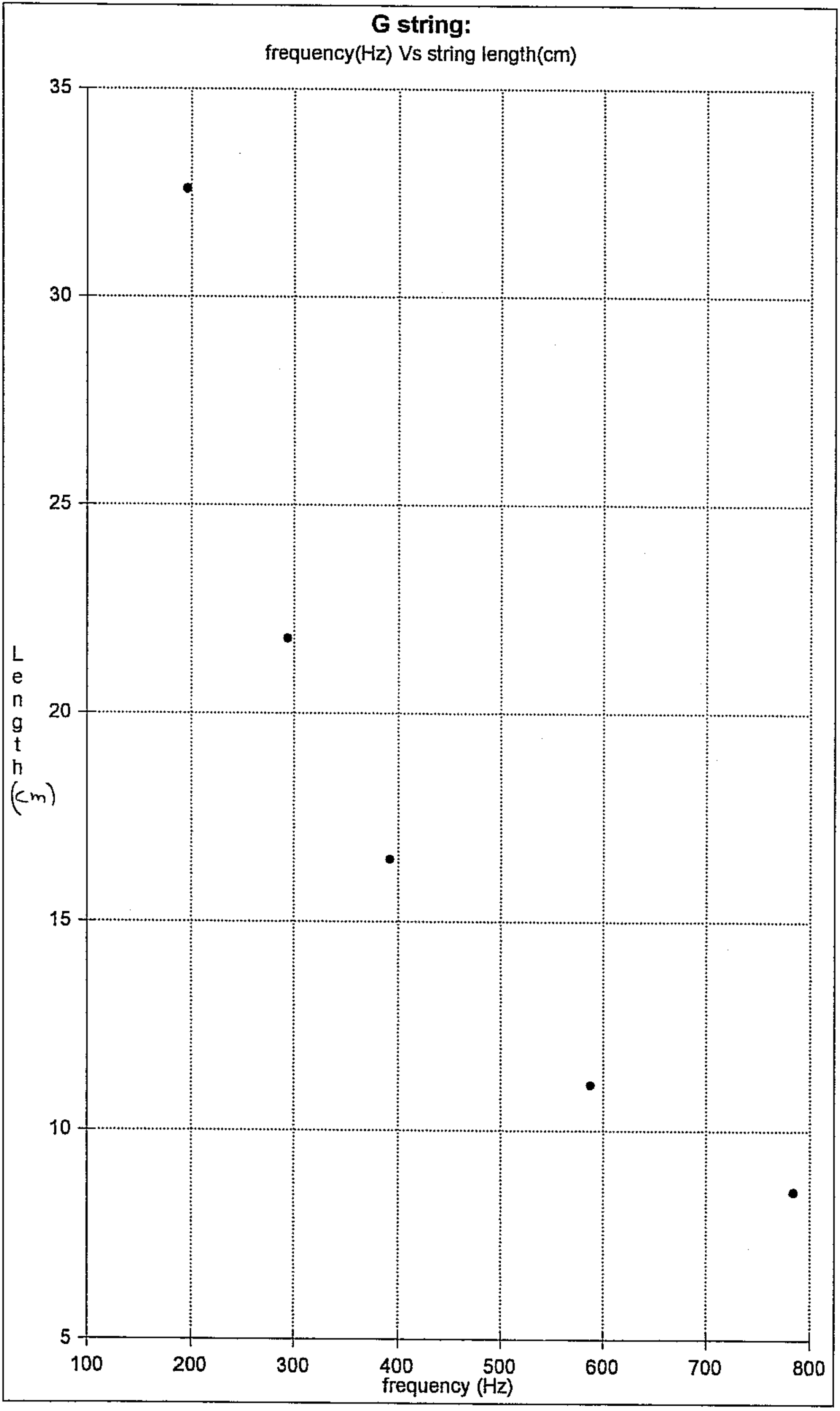
Graph 6


Note E: played on different materials
A string and E string
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
time
659.26
1318.5
frequency (Hz)
A: gut/aluminum
E: steel/aluminum

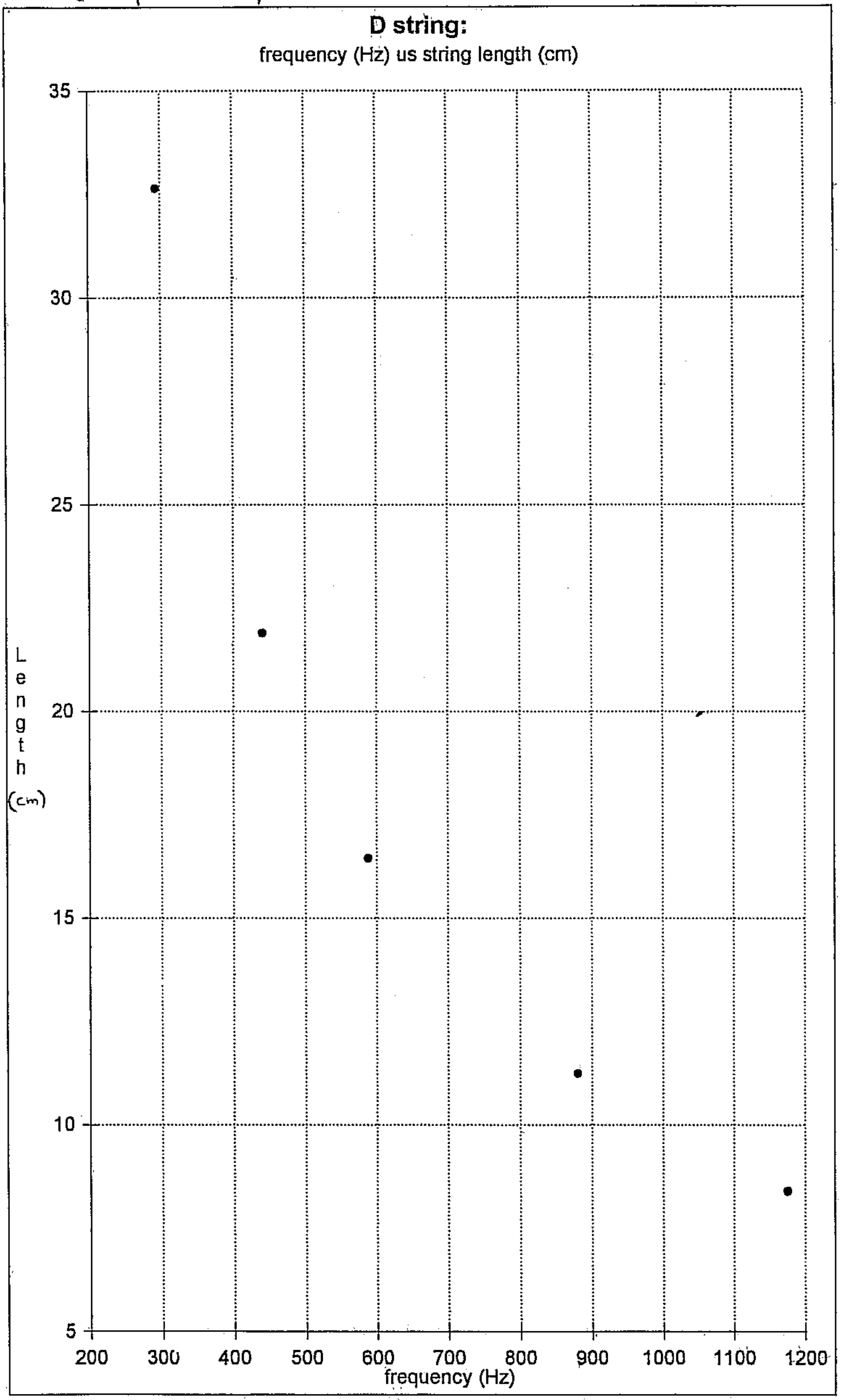
Graph 7


Note A: played on different thickness
D string and A string
time
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
440
880
frequency (Hz)
D: diameter 0.77mm
A: diameter 0.63mm

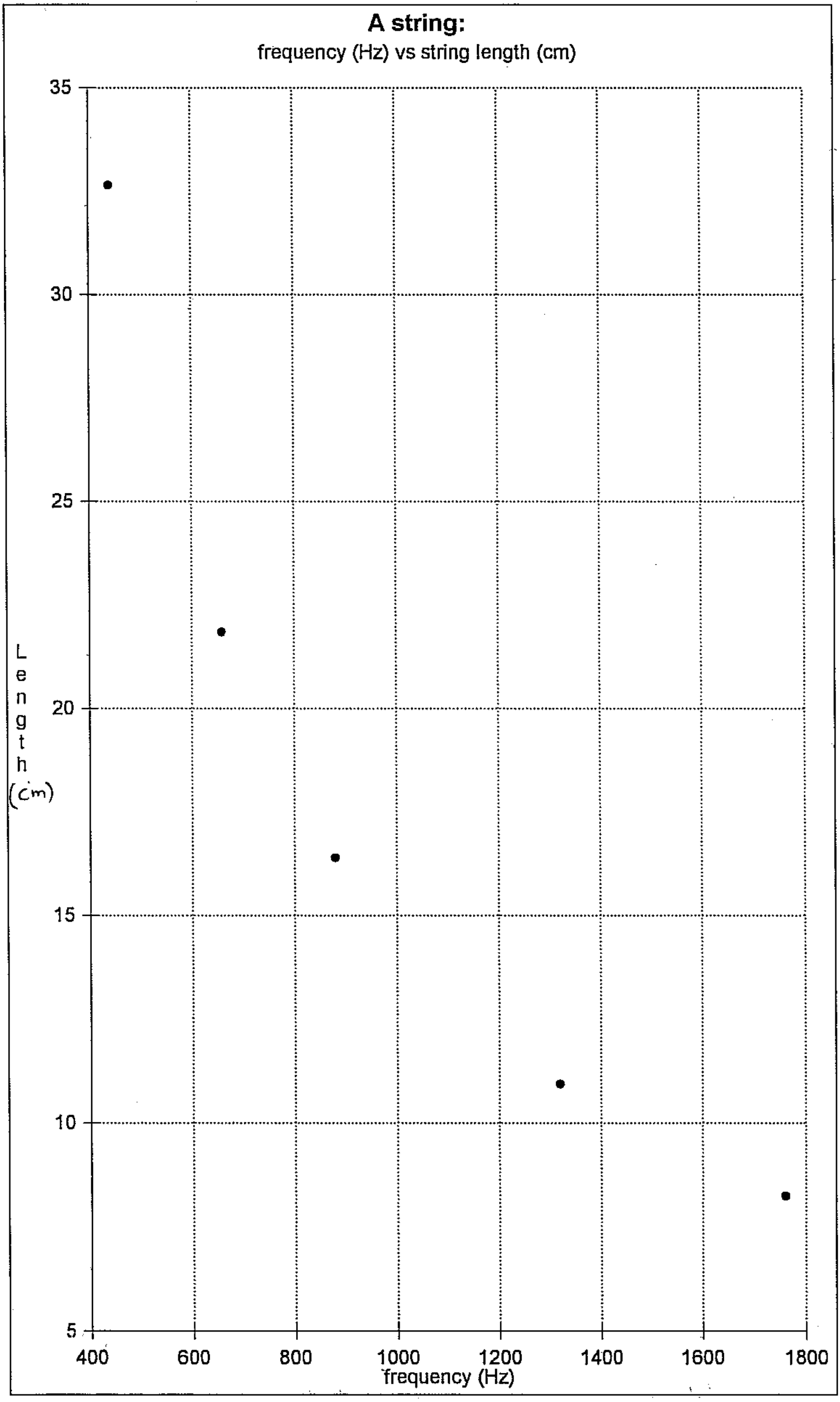
Graph 8


G string:
frequency(Hz) Vs string length(cm)
35
30
25
20
15
10
5
Length (cm)
100
200
300
400
500
600
700
800
frequency (Hz)

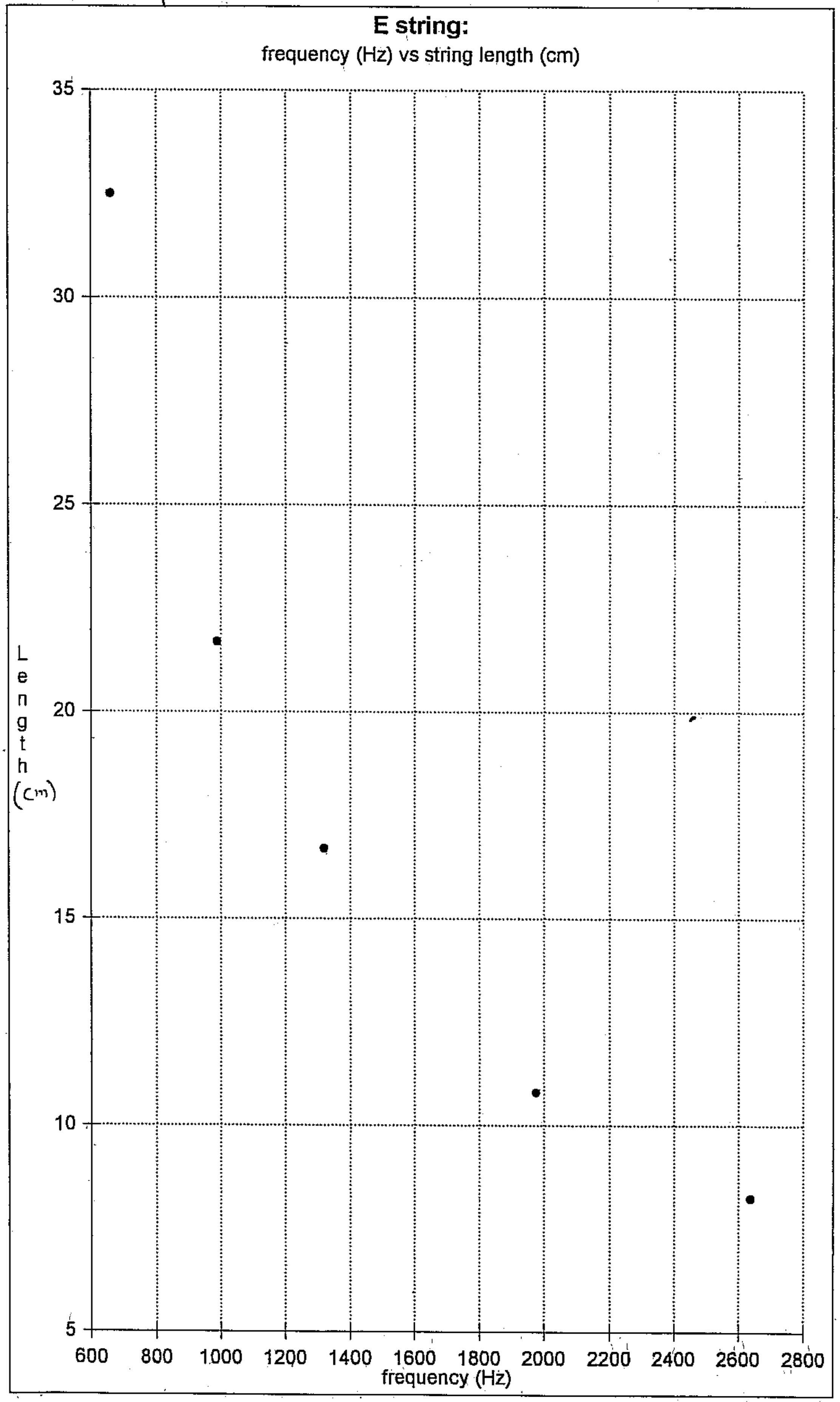
Graph 9


D string:
frequency (Hz) us string length (cm)
35
30
25
20
15
10
5
Length (cm)
200
300
400
500
600
700
800
900
1000
1100
1200
frequency (Hz)

Graph 10


A string:
frequency (Hz) vs string length (cm)
35
30
25
20
15
10
5
Length (cm)
400
600
800
1000
1200
1400
1600
1800
frequency (Hz)

Graph 11


E string:
frequency (Hz) vs string length (cm)
35
30
25
20
15
10
5
Length (cm)
600
800
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
frequency (Hz)

Errors …

1. Systematic Errors:

- Background noise in room: 15 dB. The intensity level of plucked strings were approximately 80 dB. Background noise is no more than 65 dB lower than the violin. Therefore the Signal/Noise ratio is 65 : 15 = 13 : 3

- Frequency response of the violin, the microphone and the software used: not measured.

- Length of string: ± 0.05 cm
  $$\% \text{ error} = \frac{0.05}{\textit{average length}} = \frac{0.05}{18.06} = 0.277\ \%$$

- Time measurement of software: ± 0.02 sec
  $$\% \text{ error} = \frac{0.02}{\textit{average time}} = \frac{0.02}{0.9773} = 2.046\ \%$$

2. Random Errors:

- Tuning of strings: ± 2 Hz.
  $$\% \text{ error} = \frac{2}{\textit{average frequency}} = \frac{2}{913.223} = 0.219\ \%$$

- vibrating energy loss causing shortening of sound: 0.1 sec
  $$\% \text{ error} = \frac{0.1}{\textit{average time}} = \frac{0.1}{0.9773} = 10.232\ \%$$

3. Total Error:

- in frequency
  $\% \text{ error} = 0.219\ \%$

- in length
  $\% \text{ error} = 0.277\ \%$

- in time
  $$\% \text{ error} = \frac{0.02+0.1}{0.9773} = \frac{0.12}{0.9773} = 12.28\%$$

Analysis Of Results And Observations ...

Graphs 1 to 4 were drawn relating frequency with resonance time. It was observed during the experiment, lower harmonics gave out full and colourful sounds while higher harmonics produced dull and weak sounds. For all four strings, it was found generally that the lower the frequency of note, the longer the vibration time, thus notes with higher frequencies died away faster. If lines were used to join points on the graphs, they would appear to be in the form of various exponential functions. The graphs showed the greater the fundamental frequency of string, the larger the gradient and steeper the graph. For the graph of G string, it showed the equation had a relatively small gradient. If there were more time available for this investigation, I would be able to test more frequencies and curve obtained on graph would be much smoother thus exact equations could be derived.

Graph 5 and 6 compared effect of materials used. It can be read from graphs that for frequencies 293.66Hz and 659.26Hz, the resonance periods were actually longer on less dense materials (D and E string). However for 587.33Hz and 1318.5Hz, both D and E string produced nearly the same decay time as G and A string respectively. This shows materials used (density) for strings could affect its resonance time, furthermore, I found the violin timbre was also affected by this element. Even though notes D (293.66Hz) and E (659.26Hz) died away faster on G string and A string, experienced violin players still prefer them over the other two strings, because notes on denser strings create much fuller sound thus enable the players to produce music with expressive melodies, rather than the nasal sound by stings D and E.

With different string thickness but same material (gut wound with aluminium), graph 6 shows the increase of string diameter can shorten the vibration time for both frequencies (440Hz, 880Hz). However, it was observed during the experiment that notes produced by the thicker string (D string) was richer and much more colourful than D string which created a more open, exposed sound. This is why thicker strings are preferred in slow expressive violin pieces, and thinner strings such as A and E strings are mostly used to create excitement and tension in music.

According to the hypothesis, frequencies should be inversely proportional to its vibrating length. Graphs 7 to 11 do not exactly have the answer I was expecting (a negative linear equation). The only possible explanation for this is that difference in temperature has caused density of string to change thus the ratio between frequency and length of string would be altered and graph failed to form a straight line.

In the experiment at the impulse of the pluck, an unpleasant out-of-tune effect (pitch fall) during the first fraction of a second was noticed (by ear). This occurred because the deflection of string increased and hence the average tension. However as the amplitude of the vibration died, the string length returned towards normal and pitch returns to its normal value. (refer to the equation given in the article)

If we assume the explanation is correct, the results support the theory and hypotheses.

Conclusion ...

I found that different frequencies on strings could make a significant difference in the resonance and timbre produced by violin. The loudness of sound created depended on the initial excitation and on the efficiency of the soundboard as a radiator. The time for which note sounds depended on ratio of its total initial energy to the rate at which energy is being lost. The tone quality depended on string material, plucking point and resonances of soundboard.

It was found that as the frequency gets higher, the note of the sound dies away faster and vice versa. This means when plucking, the higher the frequencies of notes produced, the greater the rate of energy loss towards air friction.

The materials and thickness used for string can affect timbre and characteristics of sound. The decay time is shorter for gut than for metal strings because of the smaller density of the string material. It normally depends on the players decision of which to use during performance.

Even though results regarding frequency and string length were not exactly accurate, equation with negative gradients were obtained. If I was to repeat the experiment, I would try to keep the temperature and humidity constant and at the same time obtain a more sensitive and accurate tuner and computer program for the investigation.

Evaluation ...

I achieved the purpose of comparing the decay time with various frequencies using different types of strings. The results obtained were mostly as expected, however those that were not were accounted for.

This investigation could be improved in several ways in order to prove the theory or hypothesis in greater detail.

After two thirds of the results had been recorded, it was realised the software had an option to amplify the sound being recorded. This would have definitely given more accurate results, had I found out earlier, but I did not have time to re-start the investigation.

If a sound proof room was available and equipment used such as the computer program and microphone could be of a higher resolution and more sensitive, the accuracy of the result can then be maximised.

Further investigation could include more types of strings, those made out of same material but different quality or same quality but compare materials such as synthetic gut with natural gut or metal. Along a slightly different path, it would be interesting to investigate how the frequency spectrum and timbre could be produce by different violin strings, and see how characteristics of a string (flexibility, tension etc.) will give a particular timbre and how it will change in quality with age.

Acknowledgments ...

Thanks to Peter for supplying me with useful computer program and Peggy for helping me with the experiment and taking results.

References ...

1. Armitage, F., et al. Physics Two, Heinemann Educational Australia, Port Melbourne, 1991.

2. Fletcher, N. Physics and Music. Heinemann Educational Australia, Richmond, 1976.

3. Johnston, I. Measured Tones, The Interplay Of Physics And Music, Adam Hilger, Bristol and New York, 1989.

Appendix 1 …

Frequency vs Vibration Time:

G string, D string, A string and E string

G String

		Trials -	time (sec)				
Note Pitch	Frequency (Hz)	1	2	3	4	5	6	Average
G	196	2.97	2.8	2.88	2.79	2.79	3.02	2.875
D	293.66	2.15	2.16	2.14	2.11	2.05	2.06	2.1116667
G	392	1	1.01	1.05	1.09	0.99	1.1	1.04
D	587.33	0.55	0.57	0.62	0.66	0.54	0.54	0.58
G	783.99	0.32	0.32	0.3	0.33	0.31	0.31	0.315

D string

		Trials -	time (sec)				
Note Pitch	Frequency (Hz)	1	2	3	4	5	6	Average
D	293.66	2.28	2.44	2.17	2.27	2.2	2.54	2.32
A	440	1.72	1.77	1.87	1.73	1.81	1.55	1.74
D	587.33	0.45	0.44	0.54	0.51	0.52	0.54	0.5
A	880	0.33	0.34	0.29	0.34	0.3	0.29	0.315
D	1174.7	0.28	0.3	0.29	0.25	0.24	0.29	0.275

A string

		Trials -	time (sec)				
Note Pitch	Frequency (Hz)	1	2	3	4	5	6	Average
A	440	1.78	1.87	1.83	1.8	1.79	1.87	1.82
E	659.26	1.47	1.42	1.39	1.42	1.59	1.4	1.45
A	880	0.32	0.34	0.34	0.36	0.32	0.32	0.33
E	1318.5	0.32	0.31	0.33	0.34	0.34	0.33	0.328
A	1760	0.31	0.33	0.31	0.3	0.3	0.32	0.32

E String

		Trials -	time (sec)				
Note Pitch	Frequency (Hz)	1	2	3	4	5	6	Average
E	659.26	1.67	1.54	1.56	1.47	1.65	1.49	1.56
B	987.77	0.69	0.73	0.73	0.73	0.75	0.7	0.72
E	1318.5	0.3	0.34	0.29	0.33	0.32	0.34	0.32
B	1975.5	0.33	0.31	0.31	0.32	0.31	0.31	0.315
E	2637	0.31	0.32	0.31	0.31	0.32	0.31	0.31

Appendix 2 ...

Frequency vs String Length:

G string, D string, A string and E string

G string

Note Pitch	Frequency (Hz)	Length Of String (cm)
G	196	32.6
D	293.66	21.8
G	392	16.5
D	587.33	11.1
G	783.99	8.55

D string

Note Pitch	Frequency (Hz)	Length Of String (cm)
D	293.66	32.65
A	440	21.9
D	587.33	16.45
A	880	11.25
D	1174.7	8.4

A string

Note Pitch	Frequency (Hz)	Length Of String (cm)
A	440	32.65
E	659.26	21.85
A	880	16.4
E	1318.5	10.95
A	1760	8.25

E string

Note Pitch	Frequency (Hz)	Length Of String (cm)
E	659.26	32.5
B	987.77	21.7
E	1318.5	16.7
B	1975.5	10.8
E	2637	8.25