Air Resistance

Contents | **Page**

This investigation proposes to investigate the air resistance of a falling object and its relationship to initial height, mass, effective surface area and final velocity of the object.

Aims:

- To show that the final velocity varies directly with the effective surface area of the object, and thus with air resistance.
- To show that the final velocity of the object does not depend on the mass of the object.
- To show that the final velocity varies directly with the initial height of the object above an arbitrary point, as does the air resistance.
- To find the acceleration caused by air resistance and thereby find the average air resistance for objects with various effective surface areas.

Theory:

Neglecting air resistance, Newton's second law gives:

$$\begin{aligned} F &= ma \\ &= mg \end{aligned}$$

where F is gravitational force (N), m is mass (kg), and g is the acceleration due to gravity ($m\ s^{-2}$).

In real life, air resistance cannot be neglected. This results in the net force becoming:

$$F = m(g - d)$$

where F is net force (N), m is mass (kg), g is acceleration due to gravity ($m\ s^{-2}$), and d is the acceleration due to air resistance ($m\ s^{-2}$). The vector component of d is in the negative direction because it is opposite to the velocity of the falling object. Dividing equation (8) by Newton's second law gives:

$$a = g - d$$

The total energy for an object that begins at rest, and falls to an arbitrary point with no air resistance is given by:

$$mgh = \tfrac{1}{2}mv^2$$

where m is the mass of the object (kg), g is the acceleration due to gravity ($m\ s^{-2}$), h is the height above the arbitrary point (m), and v is the velocity at the arbitrary point ($m\ s^{-1}$). Since mass occurs on both sides of equation (5), it can be simplified to:

$$\begin{aligned} gh &= \tfrac{1}{2}v^2 \\ v &= \sqrt{2gh} \\ &\approx 4.4\ ms^{-1} \text{ when } h = 1 \end{aligned}$$

Thus, the final velocity will vary directly with the height above the arbitrary point, with a constant acceleration of g, and not with mass. This resolves to:

$$v = 2(g-d)h$$

when air resistance is added to the equation. By measuring v and h, and taking g at its standard value it is possible to find d.

The average drag force, or air resistance, will be given by:

$$D = md$$

Drag force is also given by:

$$D = \frac{1}{2} C \rho A v^2$$

where D is the drag force (N), C is the drag coefficient, ρ is air density (kg m^{-3}), A is effective surface area (m^2) and v is velocity (m s^{-1}).[1]

Since the effective surface area varies directly with the drag force, and the drag force with the inverse square of the velocity, it is hypothesized that the final velocity will vary as the inverse square of effective surface area.

Variables:

The variables investigated were:

Mass of object (kg), initial height above an arbitrary point (m), and the effective surface area (m^2). The effective surface area was defined as the area on which air resistance antiparallel to the object's vertical velocity would act.

The variables which were assumed to be constant were:

Acceleration due to gravity taken as the given value of 9.81 m s^{-2}, [2] (In reality, this is unlikely be the case due to local variations in the gravitational field.); and air density, which would depend on the ambient temperature.

Materials:

A retort stand, boss head and clamp were used to give a convenient method of marking a constant starting point at which the object was released. Pieces of flat cardboard (corrugated) were used to provide the variations in effective surface area due to their low mass, availability and the precision with which they could be cut to specific dimensions. Plasticine was used to compose the rest of the object since it could be added in any amount to make up the required mass, using an electronic balance. The light gate and pressure sensor were set up with an electronic timer as a stopwatch, the light gate starting and the pressure sensor stopping the timer. A power supply, electrical leads, a measuring tape and a metre ruler were also used.

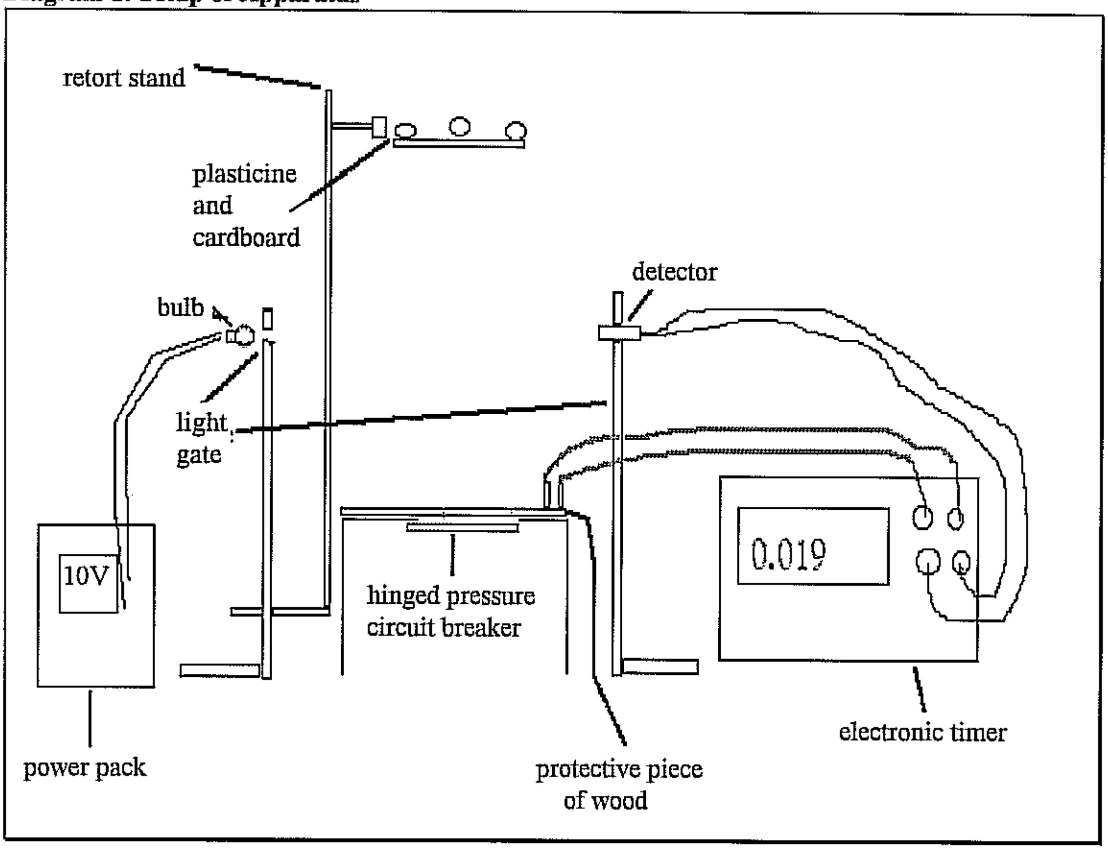
[1] Halliday et al; Fundamentals of Physics - Extended; p 148
[2] ibid. p 415

Method:

Originally, it was planned to use brass weights as the masses, since they were readily available in convenient 50 g allotments. But the pressure sensor began to disintegrate when the masses were dropped. Thus it was decided to use composite objects made of plasticine and cardboard, with smaller masses, and the sensor was reinforced with a piece of wood. (This resulted in the change in light detector-pressure sensor distance).

1. The apparatus was set up as shown, so that breaking the light beam started the timer, and hitting the pressure sensor stopped it.
   (See Diagram 1.)

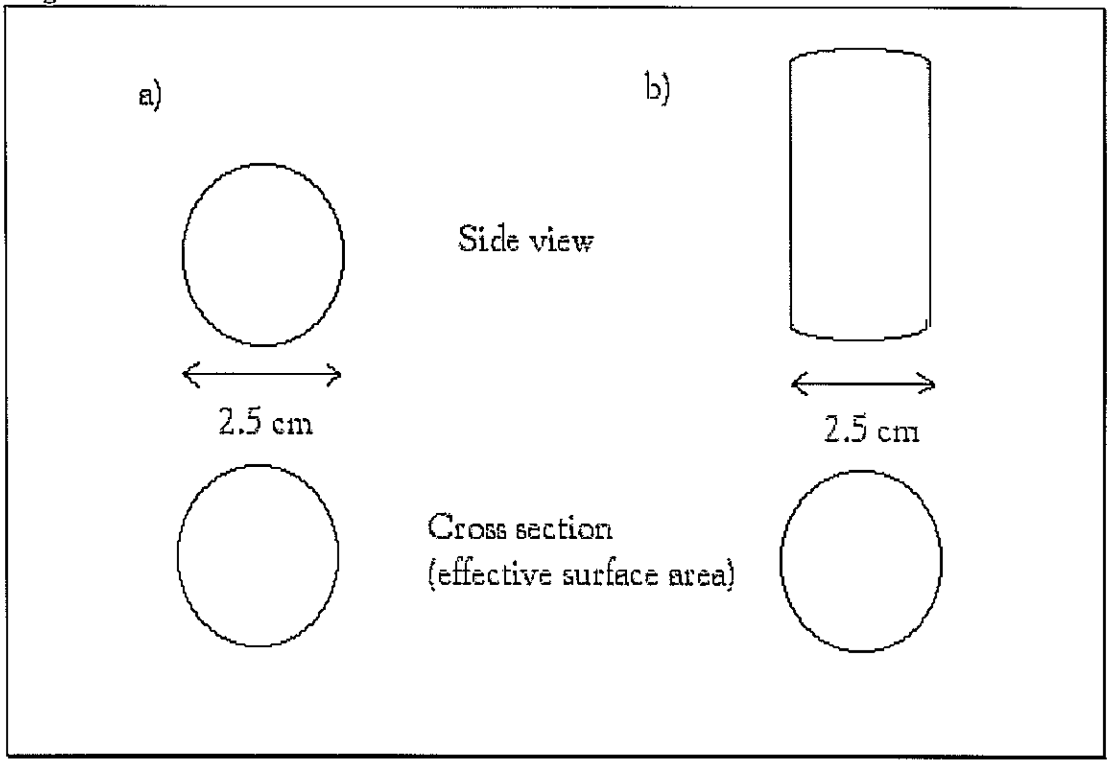
Diagram 1: Setup of Apparatus



2. The midpoint of the distance between the light beam and the top of the pressure sensor was found. The retort stand was set up so that the bottom edge of the clamp was level at 1 m above the midpoint, and the clamp was directly above, and vertically in line with both light beam and pressure sensor. The distance was defined as that from the bottom edge of the clamp to the midpoint of the distance between the light detector and the top surface of the pressure sensor.

3. A ball of plasticine of approximately 12 g in mass was rolled. (Diagram 2a)

Diagram 2: Plasticine models



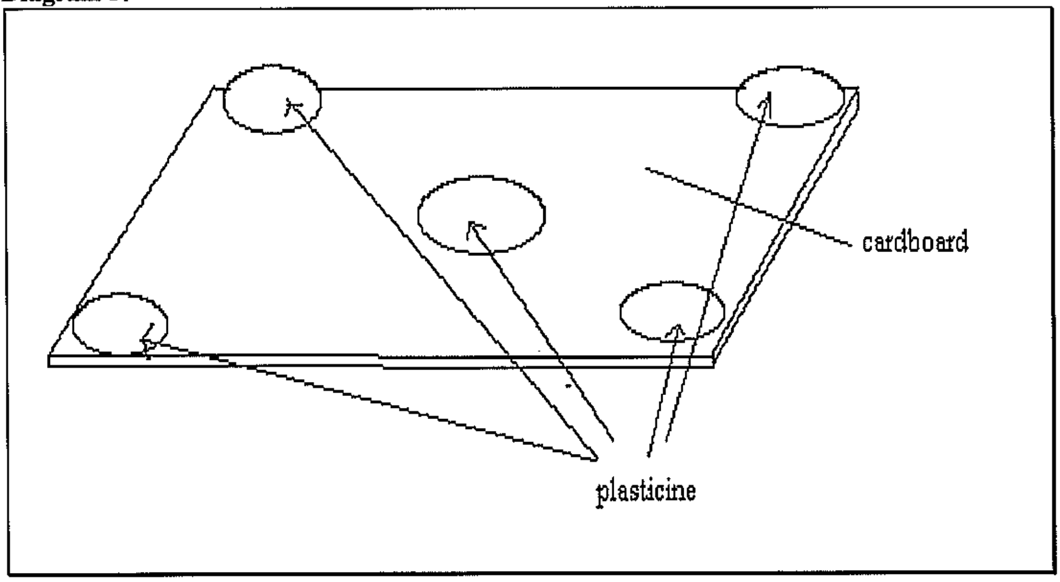
4. The ball was placed between the jaws of the clamp so that it rested level with the bottom edge of the clamp.

5. The ball was released, and the time taken for it to travel between the light beam to the pressure sensor recorded.

6. Steps 3-5 were repeated for masses of approximately 18, 24, 30 and 36 g. The plasticine was rolled into cylinders approximately equal in diameter to the first ball. The cylinders were dropped so that their ends were horizontally aligned and thus resulted in same effective surface area as the original sphere. (Diagram 2b)

7. A piece of cardboard was cut, and plasticine attached to one side.(Diagram 3) With this side uppermost, the plasticine was distributed so as to cause the cardboard to fall in a horizontal plane. The mass of the plasticine and cardboard was recorded as 70.0 g.

Diagram 3:



8. Steps 4-5 were repeated for the cardboard and plasticine, except that the cardboard was released outside the clamp due to its larger size.

9. Steps 7-8 were repeated for cardboard of varying effective surface areas. In each case the amount of plasticine was added while weighing the cardboard to give a total mass of 70.0 g. The maximum width and length of the cardboard were limited by the range of the light gate. Beyond a certain distance, the detector was unable to register the presence or absence of the light beam, thus becoming ineffectual.

10. The height of the clamp above the midpoint between light beam and pressure sensor was changed to 0.5, 0.75, 1.0, and 1.25 m. One of the cardboard and plasticine models from Step 9 was used to repeat Steps 4-5. The restriction on the maximum initial height arose from the fact that due to the large air resistance, the cardboard tended to drift sideways when falling, missing the measuring apparatus at the bottom.

Results:

Raw data is given in Appendix A together with error and uncertainties for each value.

$$\text{Velocity is calculated as } \frac{\text{distance between light gate and pressure sensor (m)}}{\text{time taken to travel between the above (s)}}$$

Varying Mass

Distance between light beam and pressure sensor = 14.4 ± 0.1 cm

Height = 1.00 ± 0.001 m

Radius = 2.5 cm

Effective surface area = $\pi r^2 = 6.25\pi\ \text{cm}^2$

Table 1: mass vs velocity

Mass (g $\pm$ 0.001)	Average Time (s x 10^{-2})	Velocity (m s^{-1})
12.15	3.2	4.5
17.75	3.1	4.6
23.75	3.2	4.5
29.97	3.1	4.6
35.78	3.1	4.6

Distance between light beam and pressure sensor changed to 13.2 ± 0.1 cm by protective wood.

Varying effective surface area

Mass = 70.00 ± 0.01 g

Height = 1.00 ± 0.001 m

Table 2: Effective surface areas tested

Object	Effective Surface Area (cm^2)	Average Time (s x 10^{-2})	Velocity (m s^{-1} x 10^1)
A	50	1.0	13
B	81	0.93	14
C	120	1.2	11
D	270	1.2	11
E	330	1.4	9.4
F	490	1.4	9.4
G	700	1.8	7.3
H	870	2.0	6.6

The final velocities were in excess of that calculated for a mass accelerated by gravity with ***no*** air resistance over the given distance (approximately 4.4 m for 1 m initial height). This was caused by the systematic error from the electronic timer. The relationship trends were not affected.

Varying Initial Height
Mass = 70.00 ± 0.01 g
Effective surface area = 870 ± 0.71% cm^2

Table 3: Initial height vs. Velocity

Height (m ± 0.001)	Average (s x 10^{-2})	Velocity (m s^{-1})
0.50	2.3	5.7
0.75	2.0	6.6
1.00	2.0	6.6
1.25	1.6	8.3

ERRORS AND OBSERVATIONS

Although the plasticine was attached to the cardboard so as to cause the cardboard to present the maximum effective surface area, that is, to fall horizontally aligned, often the cardboard would tilt or hit objects during its descent. On these occasions no result was recorded.

The electronic timer was assumed to be accurate to ± 0.001 s. This produced the largest uncertainty due to the very short times recorded, about 7%. The timer ceased to work for a period of time during the experiment, recording erratic results. After this, it was observed that the results demonstrated a systematic error, resulting in calculated velocities far in excess of those possible due to acceleration by gravity. These results were, however, consistent enough to suggest that the overall trend would be accurate.

The electronic balance was assumed accurate to four significant figures, an uncertainty of ± 0.001 g. The balance did not read to a consistent number of significant figures, depending on the magnitude of the mass. The minimum number that consistently occurred was used.

The uncertainty on the measured heights was ± 0.5 mm on each reading. This was the half scale on the ruler, beyond which it was not possible to read more accurately. Since two measurements were read, one at the top and one at the bottom, the total uncertainty was ± 0.001 m. This error was small enough to be negligible.

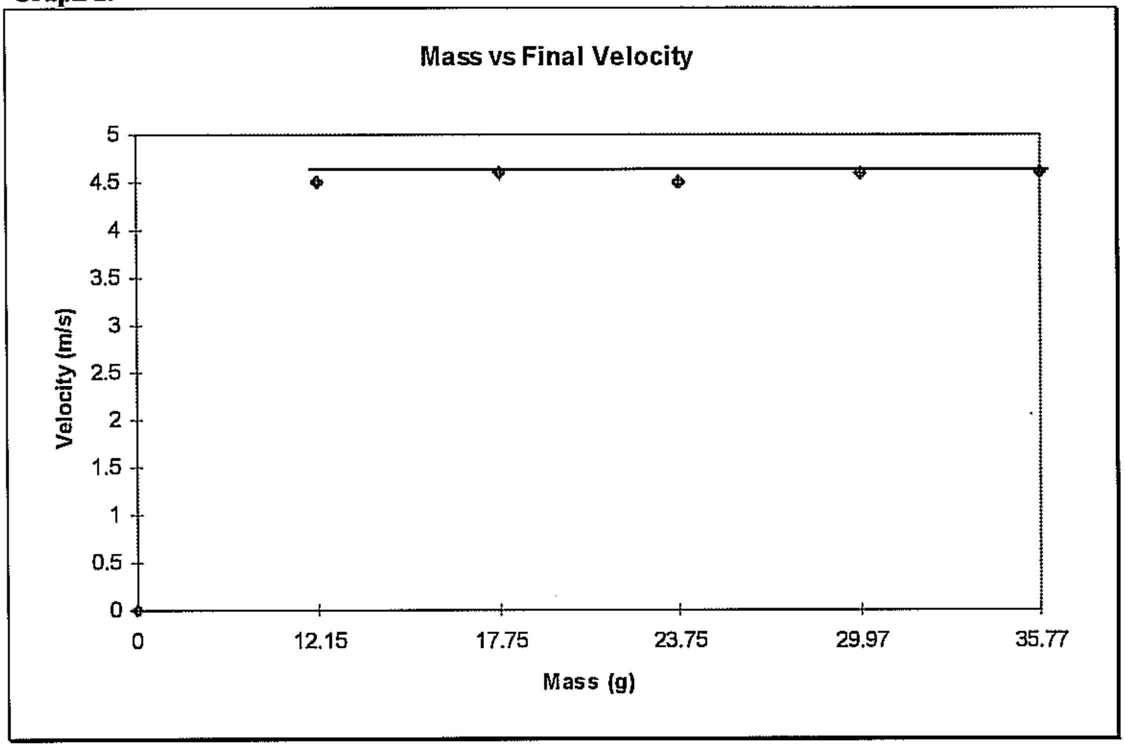
The cardboard pieces were cut using a set square, metal ruler and sharp stanley knife, giving an uncertainty of ± 0.001 m. Edge roughness may have affected the results by changing the airflow around the cardboard, but this was not investigated due to time considerations.

It was necessary to repair the pressure sensor with sticky tape as the magnets that operated the circuit breaking component became dislodged. The construction of the sensor also allowed it to bend on impact, with its supports sliding outwards and playing havoc with the carefully measured distances between sensor and light gate. In order to combat this, large masses were placed around the sensor to attempt to stabilise it.

Analysis:

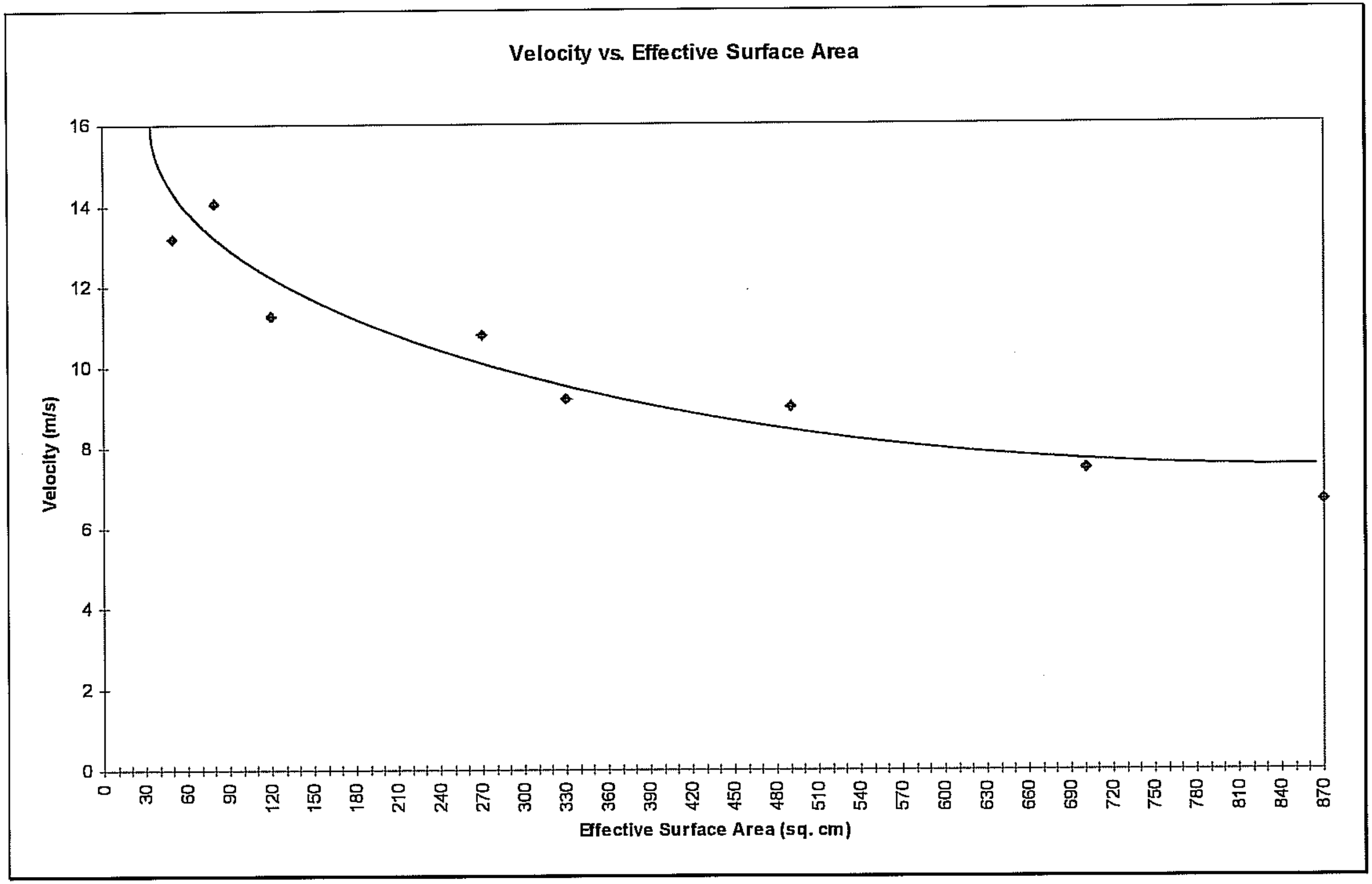
The first section of this experiment involved investigating the relationship between mass and final velocity, and thus whether there was an effect on the air resistance. The results are displayed in Graph 1.

Graph 1:



This graph clearly shows that the final velocities are not affected by the mass of the object over the range tested, supporting the hypothesis. Further testing was limited by the fragility of the pressure sensors and difficulty in obtaining larger models with the same effective surface area.

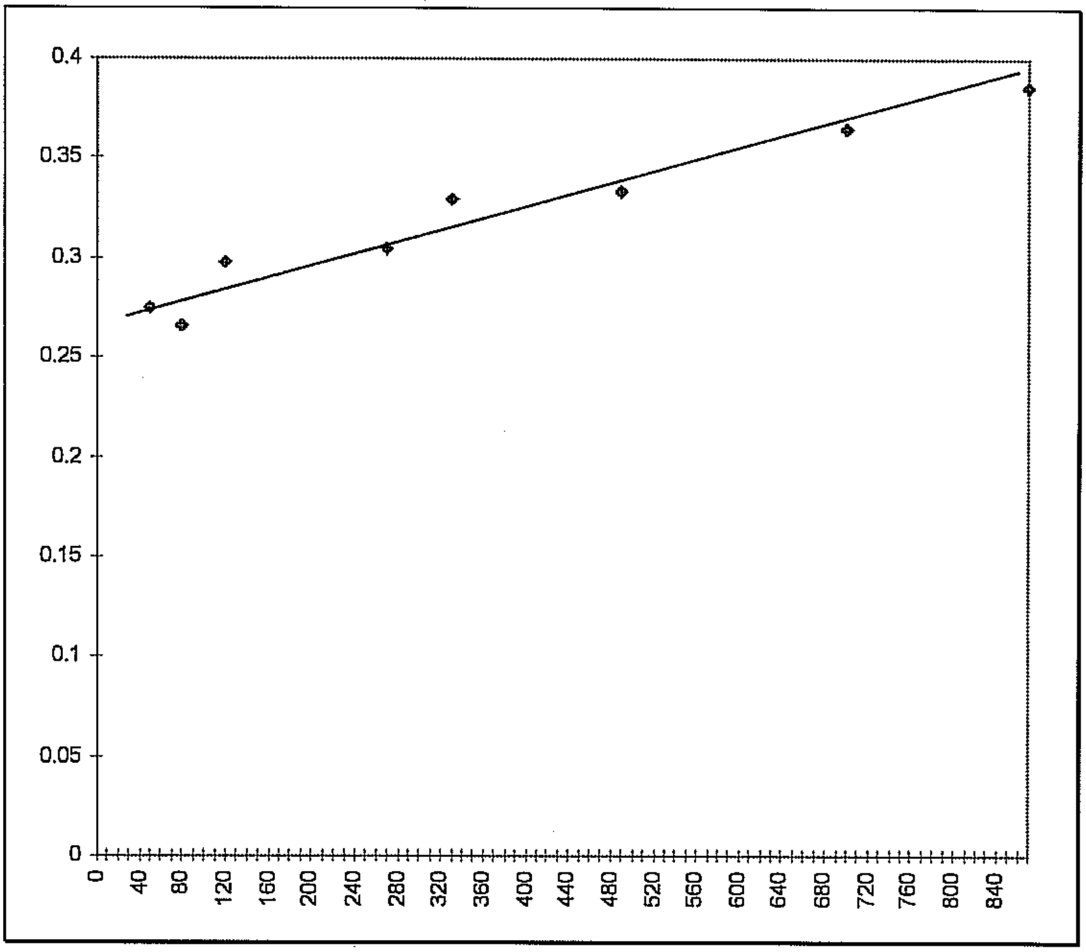
The second section of this experiment investigated the relationship between effective surface area and final velocity, and thus its effect on air resistance. The relationship between the two was graphed. (See Graph 2)



Velocity vs. Effective Surface Area
Velocity (m/s)
Effective Surface Area (sq. cm)
0
2
4
6
8
10
12
14
16
0
30
60
90
120
150
180
210
240
270
300
330
360
390
420
450
480
510
540
570
600
630
660
690
720
750
780
810
840
870

The line of best fit suggested an inverse relationship, as was predicted. However, it did not show the actual nature of the inverse relationship, i.e. inverse square/inverse cube. In order to test the hypothesis that the effective surface area varied with the inverse square of the velocity, $A \text{ vs. } \frac{1}{\sqrt{v}}$ was graphed. (See Graph 3)

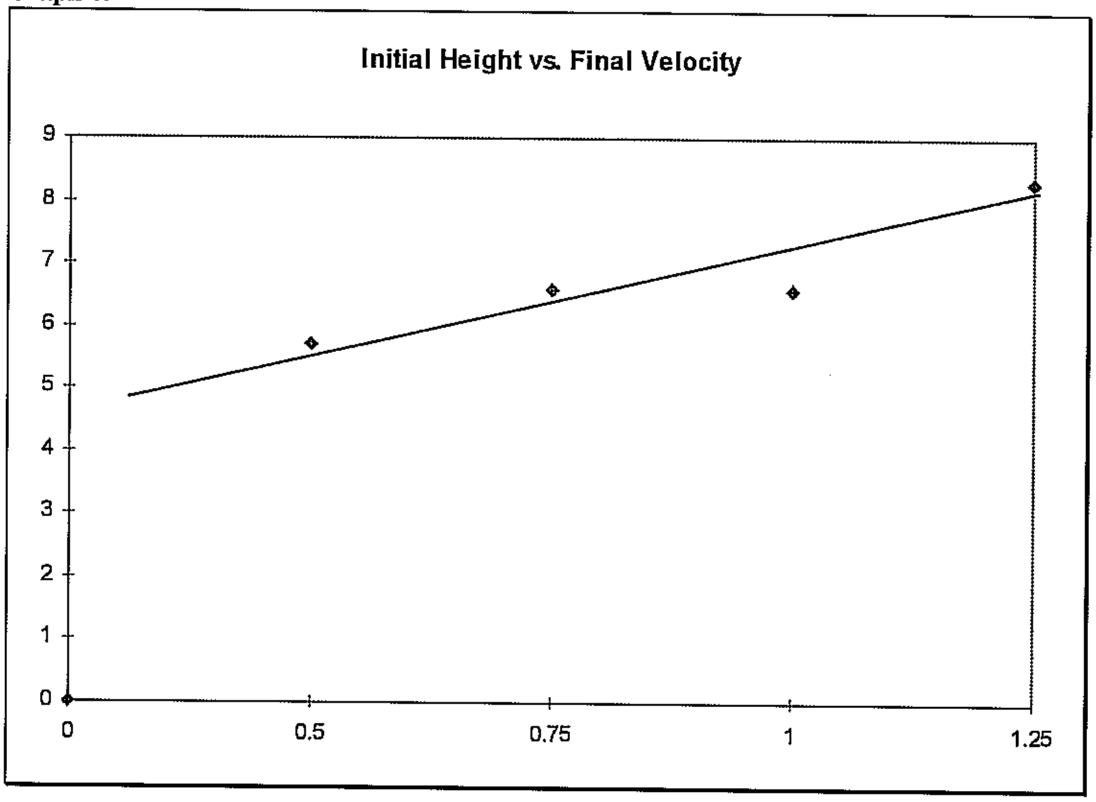
Graph 3: Effective surface area vs. $\frac{1}{\sqrt{v}}$



The linear nature of the graph supports the hypothesis that $A \, \alpha . \frac{1}{v^2}$.

The final section involved investigation of the relationship between initial height, final velocity and air resistance.

Graph 4:



The graph increases linearly, showing that v α h, as was expected.

It was not possible to calculate the average drag force and resulting acceleration because of the systematic errors in the measured final velocities.

Conclusion:

The purpose of this investigation was achieved, and the hypotheses were found to be supported within the limits of uncertainty. Thus:

1. Final velocity varies directly with original height;
2. Final velocity varies as the inverse square of effective surface area; and
3. The mass does not affect the final velocity.

Systematic errors prevented the calculation of the average drag force. The investigation should be repeated in order to eliminate this, but was constrained by time limits.

Bibliography:

Halliday and Resnick,
Fundamentals of Physics; John Wiley & Sons Inc; New York 1993; pp 148, 415

Mazzolini et al; Physics Revealing Our World; The Jacaranda Press; Milton, Queensland 1992

word count = 1940

Appendix A - Raw Data

Table 1: Mass vs. Final Velocity

Mass (g) / **Time (s)**	**12.15 ± 0.01**	**17.75 ± 0.01**	**23.75 ± 0.01**	**29.97 ± 0.01**	**35.78 ± 0.01**
1	0.032 ± 0.001	0.033 ± 0.001	0.033 ± 0.001	0.033 ± 0.001	0.027 ± 0.001
2	0.032 ± 0.001	0.031 ± 0.001	0.033 ± 0.001	0.033 ± 0.001	0.033 ± 0.001
3	0.033 ± 0.001	0.030 ± 0.001	0.031 ± 0.001	0.030 ± 0.001	0.031 ± 0.001
4	0.034 ± 0.001	0.032 ± 0.001	0.033 ± 0.001	0.033 ± 0.001	0.033 ± 0.001
5	0.028 ± 0.001	0.027 ± 0.001	0.028 ± 0.001	0.026 ± 0.001	0.030 ± 0.001
6	0.030 ± 0.001	0.033 ± 0.001	0.031 ± 0.001	0.027 ± 0.001	0.029 ± 0.001
7	0.032 ± 0.001	0.032 ± 0.001	0.033 ± 0.001	0.033 ± 0.001	0.033 ± 0.001
Average	**0.032 ± 0.001**	**0.031 ± 0.001**	**0.032 ± 0.001**	**0.031 ± 0.001**	**0.031 ± 0.001**
Velocity (m/s)	**4.5 ± 3.8%**	**4.6 ± 4.0%**	**4.5 ± 3.8%**	**4.6 ± 4.0%**	**4.6 ± 4.0%**

Due to systematic error and very large error values, the results from Test 1 were used as preliminary testing and not included in the report.

Table 2: Effective surface areas tested

Test 1			
Object	**Dimensions (cm)**	**Shape**	**Effective Surface Area (cm^2)**
A	½([12.4 ± 0.5] x [11.0 ± 0.5])	triangle	68.2 ± 8.5%
B	½([16.8 ± 0.5] + [9 ± 0.5]) x [8.0 ± 0.5]	trapezium	113.5 ± 9.6%
C	[16.3 ± 0.5] x [11.2 ± 0.5]	rectangle	182.5 ± 7.6%
D	[7.7 ±0.5] x [28.5 ± 0.5]	rectangle	219.5 ± 8.3%
E	[27.5 ± 0.5] x [10 ± 0.5]	rectangle	275.0 ± 6.8%
F	[12.7 ± 0.5] x [25 ± 0.5] + ½[15 ± 0.5]([25 ± 0.5] + [5 ± 0.5])	trapezium	542.5 ± 12.5%
Test 2			
Object	**Width (cm)**	**Length (cm)**	**Effective Surface Area (cm^2)**
A	7.8 ± 0.1	6.4 ± 0.1	50 ± 2.8%
B	7.8 ± 0.1	10.4 ± 0.1	81 ± 2.2%
C	7.0 ± 0.1	16.8 ± 0.1	120 ± 2.0%
D	23.2 ± 0.1	11.7 ± 0.1	270 ± 1.3%
E	14.9 ± 0.1	22.3 ± 0.1	330 ± 1.1%
F	21.9 ± 0.1	22.4 ± 0.1	490 ± 0.90%
G	23.6 ± 0.1	29.5 ± 0.1	700 ± 0.76%
H	22.3 ± 0.1	38.8 ± 0.1	870 ± 0.71%

Table 3: Effective surface area vs. Velocity - Test 1

Time (s)	A	B	C	D	E	F
1	0.028 ± 0.001	0.031 ± 0.001	0.036 ± 0.001	0.031 ± 0.001	0.036 ± 0.001	0.048 ± 0.001
2	0.028 ± 0.001	0.027 ± 0.001	0.038 ± 0.001	0.031 ± 0.001	0.035 ± 0.001	0.046 ± 0.001
3	0.024 ± 0.001	0.026 ± 0.001	0.032 ± 0.001	0.030 ± 0.001	0.035 ± 0.001	0.038 ± 0.001
4	0.025 ± 0.001	0.037 ± 0.001	0.034 ± 0.001	0.031 ± 0.001	0.035 ± 0.001	0.045 ± 0.001
5	0.031 ± 0.001	0.031 ± 0.001	0.031 ± 0.001	0.033 ± 0.001	0.046 ± 0.001	0.040 ± 0.001
6	0.028 ± 0.001	0.030 ± 0.001	0.036 ± 0.001	0.036 ± 0.001	0.040 ± 0.001	0.050 ± 0.001
7	0.028 ± 0.001	0.032 ± 0.001	0.033 ± 0.001	0.033 ± 0.001	0.038 ± 0.001	0.049 ± 0.001
Average (s)	0.027 ± 0.001	0.031 ± 0.001	0.034 ± 0.001	0.032 ± 0.001	0.038 ± 0.001	0.045 ± 0.001
Velocity (m/s)	5.3 ± 4.3%	4.6 ± 3.9%	4.2 ± 3.6%	4.5 ± 3.8%	3.7 ± 3.3%	3.2 ± 2.9%

Table 4: Effective surface area vs. Velocity - Test 2

Time (s)	A	B	C	D	E	F	G	H
1	0.010 ± 0.001	0.009 ± 0.001	0.012 ± 0.001	0.014 ± 0.001	0.015 ± 0.001	0.016 ± 0.001	0.017 ± 0.001	0.021 ± 0.001
2	0.010 ± 0.001	0.009 ± 0.001	0.012 ± 0.001	0.013 ± 0.001	0.014 ± 0.001	0.015 ± 0.001	0.018 ± 0.001	0.019 ± 0.001
3	0.010 ± 0.001	0.009 ± 0.001	0.011 ± 0.001	0.010 ± 0.001	0.013 ± 0.001	0.013 ± 0.001	0.018 ± 0.001	0.020 ± 0.001
4	0.010 ± 0.001	0.009 ± 0.001	0.013 ± 0.001	0.012 ± 0.001	0.014 ± 0.001	0.014 ± 0.001	0.018 ± 0.001	0.020 ± 0.001
5	0.010 ± 0.001	0.010 ± 0.001	0.012 ± 0.001	0.011 ± 0.001	0.016 ± 0.001	0.015 ± 0.001	0.016 ± 0.001	0.018 ± 0.001
6	0.010 ± 0.001	0.010 ± 0.001	0.010 ± 0.001	0.013 ± 0.001	0.014 ± 0.001	0.015 ± 0.001	0.018 ± 0.001	0.020 ± 0.001
Average (s)	0.010 ± 0.001	0.0093 ± 0.001	0.012 ± 0.001	0.012 ± 0.001	0.014 ± 0.001	0.014 ± 0.001	0.018± 0.001	0.020 ± 0.001
Velocity (m/s)	13 ± 11%	14 ± 12%	11 ± 9.1%	11 ± 9.1%	9.4 ± 7.9%	9.4 ± 7.9%	7.3 ± 6.3%	6.6 ± 5.8%