## 2 UNIT MATHEMATICS

## Curve Sketching Instructions

## Rules for curve sketching:

- Differentiating: gradient becomes the $y$-value
- Integrating: $y$-value becomes the gradient

- The parabola $y=x^{2}$ is the original function
- The line $y=x$ is the derivative
- The cubic $f(x)=1 / 3 x^{3}$ is the integral


## Differentiating:

- If the function is sloping upwards (increasing $y$-value, gradient is positive), then the derivative is above the $x$-axis.
- If the function is sloping downwards (decreasing $y$-value, gradient is negative), then the derivative is below the $x$-axis.
- If the function is at a stationary point ( $y$-value is unchanging, gradient $=0$ ), then the derivative is an x-intercept.
- If the function is a horizontal point of inflexion, the derivative is a maximum or minimum that touches the $x$-axis at its peak.
- If the function has a spike, then the derivative is undefined (it is discontinuous and should be represented by as open circle)


## Integrating:

- The original curve's $y$-values are the primitive's gradients
- If the $y$-value is positive (above the $x$-axis), the gradient of the primitive is positive
- If the $y$-value is negative (below the $x$-axis), the gradient of the primitive is negative
- If the $y$-value $=0$ (on the $x$-axis), the primitive curve has a maximum or minimum turning point or a horizontal point or inflexion
- Note: If the curve approaches a value of 1 at infinity, the primitive approaches a gradient of 1 at infinity, so a horizontal asymptote is created at this point with a gradient of 1 (the asymptote has this gradient but the primitive function never reaches it). The gradient of the asymptote in this situation would be $\mathrm{y}=\mathrm{x}$, though other situations can exist.
- If it passes through the $x$-axis from positive to negative, then the primitive is maximum at that point
- If it passes the $x$-axis from negative to positive, it is a minimum at that point
- If it is a maximum or minimum that touches the $x$-axis at its peak, then it is a horizontal point of inflection
- If there is a discontinuous point in the original function, then the primitive will also have a discontinuous point or a spike
- Note: When drawing it is helpful to divide the curve into vertical strips with a demarcation through each intercept, stationary point, inflexion point and extrema

